USING COOPERATIVE LEARNING IN A GRADE 11 CLASSROOM TO ENHANCE CONCEPTUAL UNDERSTANDING OF TRIGONOMETRY

by

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Using cooperative learning in a grade 11 classroom to enhance conceptual understanding of trigonometry

I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I submitted the dissertation to originality checking software and that it falls within the accepted requirements for originality.

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ABSTRACT

This study employed a qualitative approach to investigate the use of cooperative learning to enhance conceptual understanding of trigonometry in a Grade 11 mathematics classroom, conducted at a high school in Moletlane Circuit, Capricorn District in Limpopo Province, South Africa. A single case study was used as a research design to get an in-depth analysis and collect detailed data using semi-interviews and lesson observation of the cooperative learning of trigonometry in Grade 11 from the learners and the teacher. Participants were purposely chosen and consisted of (n=30) Grade 11 mathematics learners and their mathematics teacher. Data from the participants were collected through semi-structured interviews and observation, with the aid of observation guide (Appendix C) for three weeks.

The salient findings from the study showed how cooperative learning was used, research questions, the approaches, the teacher did not adequately highlight the importance of trigonometry to students without integrating the topic to real-life situations. Some students said that the teacher did not teach trigonometry in a manner that they understood, which made trigonometry challenging for them. Concerning cooperative learning, the study found that many learners were passively engaged, listened to or watched the teacher. Mainly, the study recommends teacher-training institutions to host practical workshops to help teachers integrate theoretical training and practical cooperative learning experience. While this study was qualitative in nature, future researchers could conduct quantitative data collection. This would allow for the collection of numerical findings through survey questionnaires.

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LIST OF ABBREVIATIONS

CAPS	Curriculum Assessment Policy Statement
CL	Cooperative Learning
FET	Further Education and Training
IMPROVE	Introducing new concepts, Metacognition question strategy, Practicing, Reviewing and reducing, Obtaining mastery, Verification and Enrichment
NSC	National Senior Certificate
SMK	Subject Matter Knowledge
STAD	Student Teams-Achievement Divisions

KEY TERMS

Cooperative learning, Trigonometry, Constructivism, Curriculum, Pedagogy, Grade 11 mathematics learners, enhance, conceptual understanding

CHAPTER ONE

INTRODUCTION AND BACKGROUND

1. INTRODUCTION

Mathematics is the most important subject for the world's economic growth at large since many educational opportunities and jobs require a high level of mathematics knowledge (Kilpatrick, Swafford, & Findell, 2001). Mathematics has been one of the most underperformed subjects in South Africa according to the National Senior Certificate (NSC) diagnostic report (Diagnostic Report, 2017). Trigonometry is a topic in mathematics that most students and teachers believe it to be difficult compared to other topics of mathematics (Gür, 2009). Gillies (2016) further argues that high performing students grasp trigonometry conceptually because lower performers find it too difficult to understand. Unfortunately, many learners (75.1%) are underperforming in trigonometry (NCS Diagnostic Report, 2017). The literature shows that methods like cooperative learning could be useful in improving the teaching of this concept to enhance conceptual understanding (NCS Diagnostic Report, 2017). This study argues that cooperative learning may help alleviate this challenge.

Researchers acknowledge that Cooperative Learning (CL) is an international pedagogical practice that makes learners active in the process of learning from pre-school to tertiary level in different subject areas (Gillies, 2016). Furthermore, CL is viewed as a well-structured teaching strategy that produces more positive performance than innovative curriculum textbooks or the use of technology in reading and mathematics (Slavin, 2003). The use of curriculum textbooks for the process of teaching and learning has been the traditional method for teachers in classrooms (Bot & Eze, 2016 and Bialangi, Zubaidah, Amin & Gofur, 2015). Hohensee (2016 argues that this method has been limiting learners' ability to explore their skills to solve mathematical problems conceptually through social interaction. Also, Weber, Knott, and Evitts (2008) consider technology to be limiting social interaction among learners because it relegates them into silos where their peers are essentially replaced by electronic devices.

An educator’s utmost challenge in teaching trigonometry is finding the most effective pedagogy for their students (Gür, 2009). Teachers must get assistance to design effective curriculum and determine how best students learn by understanding and assessing students’ involvement in learning trigonometry (Tsay & Brady, 2010). Educators may hesitate using cooperative learning in a mathematical classroom because they have inadequate experience and knowledge of using cooperative learning as a teaching strategy (Gür, 2009).

Vygotsky’s (1992) theory of scaffolding supports social interaction, stating that learners can learn more information quickly than they could with traditional instruction. Furthermore, learners can solve mathematical problems cooperatively before solving the same problem on their own. It is for this reason that this study investigated cooperative learning as a way to improve student understanding of trigonometry.

1.1 Key Concepts

Cooperative learning is a teaching method where students of mixed levels of ability are arranged into groups and rewarded according to the group's success, rather than the success of an individual member.

Trigonometry is a branch of mathematics that studies relationships between side lengths and angles of triangles.

Constructivism is a theory that suggests that humans construct knowledge and meaning from their experiences.

Curriculum describes the subjects comprising a course of study in a school or college.

Pedagogy is the method and practice of teaching, especially as an academic subject or theoretical concept.

1.2 STATEMENT OF THE PROBLEM

Weber, Knott, and Evitts (2008) view trigonometry as one of the most important topics in high school mathematics, because it links algebra, geometry and graphics. This connection of different topics by trigonometry makes it conceptual in nature (Orhun, 2001; Brown, 2005; Challenger, 2009). However, instructional strategies that teachers use during trigonometry classes are procedural, not conceptual (Orhun, 2001).

For example, learners were asked to find the value of x if $\sin 30 = x$. Most learners got the answer wrong because they could not understand the maximal domain of the sin function. In another instance, learners had an activity to find the measure of a central angle q if an arc of the length $p/3$ in radian, most students got it wrong because their knowledge of the concept of measure of a central angle in radians was inadequate.

Unfortunately, these strategies are found to be reasons behind learners' demonstration of mistakes and misconceptions when solving trigonometric problems, resulting in poor performance in assessments (Orhun, 2001). Orhun further claimed that learners had difficulties in understanding the connection between the measure of any angle in degrees and in radians.

Amongst instructional strategies that are predictors of positive performance is cooperative learning (Tsay & Brady, 2010). Jansen (2012) also argues that cooperative learning can "promote conceptual understanding of mathematics development of mathematical reasoning skills in addition to procedural fluency" (p.38). Cooperative learning environments, where learners are conceptually orientated, contribute towards learner's ability, skill and attitude in solving mathematical problems of trigonometry (Kilpatrick, Swafford & Findell, 2001).

Thus, studies involving the teaching of trigonometry to enhance its conceptual understanding are needed to provide more clarity on how learners can learn trigonometry for understanding.

1.3. RATIONALE FOR THE STUDY

My experience of cooperative learning in a mathematics classroom, developed through three phases: as a learner in high school, as a student at university and as a mathematics teacher.

The understanding of mathematics has been a challenge for most learners in high school, the Grade 12's in particular. Most learners would think they understand the mathematics while the teacher is in the front teaching and solving problems, but when they had to answer individual assessment tasks, they struggled and often failed. This is when classmates would seek help from the ones whom they regarded as understanding the mathematics better. They would go to that learner's table for assistance and soon other learners would join the discussion. The spontaneous, informal grouping process intensified, particularly in the panic towards the final examination preparation. These natural groups were beneficial, however it was too late to help those learners with deep rooted challenges, to pass the examination. Although their performance and understanding improved, they regretted having joined late.

At university, the pattern repeated. Bright students were joined by one student who needed help with assignments preparations, followed by other students. During test- and examination preparations and assignment writing, students would meet to discuss and prepare the tasks ahead. I found myself at the epi-centre of one such group, noticing the beneficial effect of the group approach for students who returned to the group throughout the year, to discuss assignments and tests preparations in the first trimester and examination preparations towards the end of the second semester. The informal grouping was beneficial to myself too as I managed to land a distinction in mathematics education in my final year.

In school and at university, not everybody was keen to join the informal mathematics groups though, although the beneficial effect was obvious. I can only speculate as to why they preferred not to join - could have been personality, individuality or pride of getting help from their peers play a part? My conclusion, entering into the teaching of mathematics, was that group work was beneficial, and if these groups could be formalised, more learners could gain from it.

As a Grade 12 teacher, I looked at the history of the learners' performance in mathematics for Grade 10 and 11 and I was seriously concerned: most learners have not passed mathematics in both grades, but they have been progressed to the next grade since the pass requirements permitted such progression. After several lessons, I identified four learners with great potential and quick understanding and I made each learner a group leader, whom I grouped with average and low performing learners. This class of matric passed with 76% in mathematics, while the previous year's performance was 15%.

The teaching and learning of mathematics in senior and further education training (FET) phase is mainly teacher-centred, and that makes learning passive. While there are positive results from traditional approaches to teaching trigonometry, teachers may also benefit from a method that makes learning active (Sezer, 2010). The advantage of traditional approaches to teaching trigonometry is that learners can be able to reproduce the lesson taught by the teacher to solve mathematical problems. While it is important for teachers to instill procedural fluency on learners, the classroom environment can also benefit from a conceptual approach to teaching mathematics (Sezer, 2010). A classroom that explores a conceptual approach to teaching trigonometry can ensure that learners are competent when applying mathematics knowledge to the real world because the teacher-centred method limits their problem-solving skills. This is an additional advantage on top of the procedural fluency received from traditional approaches to teaching trigonometry (Boaler, 2008).

Research has argued that knowledge of teachers and the support material used in traditional teaching approaches is insufficient in trigonometry, among other teaching subjects (Motsheka, 2012). Therefore, educators' greatest challenge is to determine the most effective teaching strategies for their students (Tsay & Brady, 2010). In my experience, teachers have adequate experience and knowledge for teaching mathematics (trigonometry) though their teaching strategy is monotonous: traditional classroom set-up with all learners listening to the teacher.

To increase learners' success and conceptual understanding, educators must revise the way they teach mathematics in schools (Ke & Grabowski, 2007). I therefore decided to explore cooperative learning as one method that could improve the teaching of trigonometry. This was motivated by the aspect of cooperative learning that allows learners to be actively involved in the process of learning mathematics.

The teaching methods used by teachers in a mathematics classroom should accommodate all the learners with the goal of learners conceptually understanding the topic and succeeding during an assessment.

Problem-solving in mathematics is predominant, learners do not need to be taught to memorise formulas but to make connections between mathematical ideas (Kilpatrick et al., 2001). Cooperative learning can help learners make connections as it provides an environment for learners to discuss, justify, investigate and challenge their ideas. It also allows learners to communicate mathematical ideas with one another (Flynn, 2013).

1.4 PURPOSE OF THE STUDY

The purpose of this study was to use cooperative learning to enhance conceptual understanding of trigonometry in Grade 11 mathematics classroom.

Research questions

This study had the following research questions:

- What are teachers' views of cooperative learning in the classroom?
- What are the cooperative learning opportunities for students in trigonometry?
- What is the effect of cooperative learning on students?
- What cooperative learning strategies can help improve students' competence in trigonometry?

1.5 RESEARCH METHODOLOGY

1.5.1 Research design

A research design is a basic plan that guides the data collection and analysis processes of conducting research. The study used case study as a research design because scholars such as McMillan (2012) argue that it offers an in-depth analysis of one or more events, settings, programs, social groups, or individuals in their natural setting. Moreover, the research design was chosen because, according to Yin (2012), case studies are bounded by time and activity, and researchers collect detailed information using a variety of data collection techniques over a sustainable period.

1.5.2 Research paradigm

According to MacNaughton, Rolfe & Siraj-Blatchford (2001), a research paradigm is a belief about the nature of knowledge, a methodology and criteria for validity within which research takes place. In this study, the interpretive paradigm was employed to understand the experiences of students under the use of cooperative learning in a trigonometry classroom. Creswell (2003) is of the view that the interpretive paradigm is appropriate for such studies because, in this case, it helps with the understanding of the perceptions of learners through their experiences within a cooperative learning environment. Therefore, this study used the learners' responses to cooperative learning to construct and interpret their understanding from the collected data (Cao Thanh & Le Thanh, 2015).

1.5.3 Research approach

The research approach used in this study was a qualitative research methodology. Qualitative research is an approach for discovering and understanding the meaning individuals or groups attribute to a social problem (Creswell, 2014). This study used a qualitative research approach to understand the meanings constructed by students - how students make sense of their own experiences and knowledge they have about trigonometry (Merriam, 2009:13). According to Creswell (2014), the qualitative research approach involves emerging questions and procedures data in this approach is collected in the participants' setting.

1.5.4 Sampling

The study was conducted in Limpopo Province, Capricorn District in one school in the Moletlane Circuit. Purposive sampling technique was used to select participants in this research. The study's focus on cooperative learning used small groups of six students in Grade 11 mathematics classrooms.

1.5.5 Data Collection

The data were collected through a semi-structured interview and lesson class observation. Interview questions were designed for the participants as well as an observation guide to help form a record of important information. Snapshots were randomly taken throughout the lesson on learners' written activities and activity tasks shared on the board given by their teacher during the lesson. The interview and observation guide was closed-ended for only gathering data on the teachers' understanding of cooperative learning in the classroom as well as to understand how teachers provide opportunities for cooperative learning in teaching trigonometry. The semi-structured interview and observation guide aimed to understand how learners benefit when cooperative learning is used in a trigonometry classroom. It also sought to find out the strategies that teachers employed to enhance cooperative learning competence in teaching trigonometry.

1.5.6 Data Analysis

The data were analysed using thematic analysis. According to Braun & Clarker (2006), thematic analysis is the process of identifying patterns or themes within qualitative data. The researcher organised collected data from interviews and notes taken during the lesson observation and searched for patterns and themes collected and then transcribed data recorded during interviews (Flick, 2013).

The thematic analysis enabled the researcher to report the experiences of the participants gathered during the data collection process. To analyse, snapshots, transcripts from the interview and notes taken during observation were thoroughly read by the researcher. Relevant phrases or sentences were labelled to search for themes with broader patterns of meaning and review the themes to make sure that they fit in the data. The categories were labelled and decisions were made thereafter with regards to relevance and how they were connected. The categories and connections constituted the main results of the study.

1.6 SIGNIFICANCE OF THE STUDY

The study seeks to help educators with implementing cooperative learning in teaching trigonometry successfully. This study needs to be be conducted so as to inform teachers about the effective use of cooperative learning of trigonometric mathematical concepts. The argument of this study is that while traditional teaching methods empower learners with the procedural fluency, strategies such as cooperative learning in a mathematical classroom can provide additional benefits by allowing learners to develop their mathematical reasoning and creative skills in preparation for more abstract mathematics in tertiary institutions (Grade 10-12 NSC, Curriculum Assessment Policy Statement (CAPS), 2012).

Theoretically, the study contributes to the existing body of knowledge on the benefits of using cooperative learning as the prominent pedagogy to enhance conceptual understanding of trigonometry. Using case study improved the understanding of the problem, with the intent of contributing to the solution (Drislane, 2011).

1.7 LIMITATIONS OF THE STUDY

This study extracted a sample of students and a teacher. Although the sample was enough to answer research questions posed herein, the findings from this study cannot be generalised to Grade 11 learners and teachers in South Africa.

The study used a qualitative research design and semi-structured open-ended qualitative interviews and observations. Thus, a worthwhile direction for future research would be using a quantitative research design or mixed methods with curriculum advisors, principals and more teachers to provide a more in-depth understanding of the use of cooperative learning in learning trigonometry.

1.8 DELIMITATION

The delimitation of this study was that the researcher only included written group tasks, structured questions and observations rather than additional open-ended responses, which might have allowed some respondents to contribute in-depth information. The study exclusively focused on trigonometry and did not consider other mathematical areas.

1.9 CHAPTER OUTLINE

Chapter 1 provided an introduction to and background of the study. A problem statement and core research questions, the purpose of the study and the significance of the study were outlined.

Chapter 2 reviewed the literature relevant to the study. The section first introduced the conceptual understanding of trigonometry, then explained the theoretical understandings of cooperative learning. The history of cooperative learning was outlined as well as some of the perceived benefits of cooperative learning. Additionally, the chapter explained the perceived effects of applying cooperative learning in teaching trigonometry and concluded with a discussion of the theoretical framework that underpinned the study.

Chapter 3 presented the research methodology, focusing on the research design, methodology choice, research strategy, research approach, research site, population, sampling, data collection method, and thematic analysis.

Chapter 4 discussed the analysis and presentation of the study results. The chapter included the themes that recurred in the thematic data analysis and summarised the findings.

Chapter 5 featured the summary of findings, made recommendations and concluded the discussion of the results. Literature findings, as well as the theoretical underpinnings of the study, were used to interpret and summarise the thematic findings from the study.

1.10 CONCLUSION

This chapter introduced the topic under which the study was investigated. A brief background to the study was provided and the statement of the problem was highlighted. The section further delineated on the purpose of the study, the research questions and the significance of the study. The next section reviews the literature relevant to the study.

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1. INTRODUCTION

A literature review is important for describing how proposed research is related to prior research in terms of findings. It shows the originality of research, its relevance as and the gap in the literature that the research intends to address. For that reason, this chapter discusses the conceptual understanding of trigonometry and the history of cooperative learning. It also discourses on the tenets or principles that underpin cooperative learning and the vital success factors of cooperative learning in trigonometry. This section further outlines the literature on the perceived benefits of cooperative learning as well as the supposed impact of cooperative learning. Moreover, this chapter reviews some literature on the experiences of learners who have been exposed to cooperative learning and incorporate essential feedback from teachers.

2.2. CONCEPTUAL UNDERSTADNING OF TRIGONOMETRY

Conceptual understanding is a comprehension of mathematical concepts, operations and relations (Kilpatrick et al., 2001). Students with conceptual understanding comprehend why a mathematical idea is important and kinds of contexts in which it is useful (Kilpatrick et al., 2001). Conceptual understanding and mathematical skills have an effective role/impact on learners solving mathematics problems correctly (Gultepe, Celik & Kilic, 2013).

Some schools of thought define trigonometry as “a study of triangles, the relationship between their sides and angles, the functions of sine, cosine, tangent, cotangent, secant and cosecant, along with the graphs of these functions” (Walsh, Fitzmaurice, & O'Donoghue, 2017). The conceptual understanding of trigonometry is difficult for many learners as it results in poor mathematical performance (NCS Diagnostic Report, 2017).

Trigonometry is broad. It includes, for example, trigonometric functions, 2-D and 3-D problems, basic ratios, reduction formulae and expansion, and general solution. This study focused on basic ratio and reduction formulae because these were the most failed sections in trigonometry (NSC National Diagnostic Report, 2013). Achievement in NSC mathematics matric results was 51.1% and 51.9% in 2016 and 2017 respectively, and trigonometry had three questions in 2017 with 43% on average.

Trigonometry is one of the branches of general mathematics in the secondary schools in South Africa that deals with the study of triangles and the relationships that exist between the sides and angles of triangles (Bot & Eze, 2016). The knowledge of trigonometry is important; it is useful in studying different courses in tertiary institutions like engineering, physics, architecture and survey. The knowledge of trigonometry is also imperative for uniting geometry and algebra that are studied by all learners in secondary school in South Africa (Lassa, 2012).

2.3 CHALLENGES WITH TRIGONOMETRY

The common errors and misconceptions exhibited by learners in solving trigonometry problems are, among others, the incorrect substitution e.g. question 5.2.3 $CosB = 1 - sin^2B$ was substituted as $Cos2B = 1 - Sin^2(\frac{-5}{\sqrt{34}})^2$ instead of $Cos2B = 1 - (\frac{-5}{\sqrt{34}})^2$ and in question 5.1. Learners struggle with reduction formulae, especially with the signs of the reduced trigonometric ratios, while some students could not apply co-functions correctly (National Senior Certificate Diagnostic Report, 2017).

Algorithms should be used in algebraic equations and learners should be encouraged to use them, but they should be developed and used in parallel with conceptual knowledge (Gultepe, et al., 2013). Conceptual understanding is the ability to solve both lower and higher order trigonometric problems correctly, being able to make connections of previous lesson to apply the acquired knowledge in various aspects of trigonometric tasks. When a learner can proof the following identity$sin^2x + cos^2x = 1$, then the learner can apply the knowledge to calculate $sin^2(43^{\circ}) + cos^2(43^{\circ})$.

Researchers such as Orhun (2002) studied the difficulties faced by students in using trigonometry when solving problems. Orhun found that the students did not develop the concepts of trigonometry clearly. They made some mistakes and the teacher-active method and memorising methods enhanced students' knowledge of trigonometry only for a moment. This proves to be problematic because students do not retain knowledge eventually.

Similarly, Brown (2006) studied students' understanding of sine and cosine. She reached a fragment called trigonometric connection. Brown's study showed that learners had an incomplete and disjointed understanding of the three major to view sine and cosine as the coordinates of a point on the unit circle, as a horizontal and vertical distance that are graphical entailments of those coordinates, and as ratios of sides of a reference triangle (p.228).

Furthermore, Orhun's (2002) findings focused on the mistakes students made and the lack of the development of trigonometric concepts by learners. He further explained how the method used (teacher-active) limited the students' knowledge, and how it affected their memory span. On the other hand, Brown (2006) also learned that learners' understanding of sine and cosine was not complete.

This study notes a gap, drawing from Brown and Orhun's findings, in the pedagogy or teaching method teachers use to assure students' in-depth understanding and memory retention of knowledge. Hence, the researcher believes that approaches such as cooperative learning may enhance conceptual understanding, including memory retention, making connections and assure in-depth understanding of trigonometry.

Researchers such as Kamber and Takaci (2018) investigated the problematic aspects encountered by high school learners in learning trigonometry. Their study was based on making sense of mathematics through perception, operation and reasoning in the case of trigonometry. In their study they analysed students' understanding of trigonometric concepts in the structure of triangle and circle trigonometry context. The study by Kamber and Takaci (2018) examined two groups of high school students a) beginners of trigonometry in high school (17 years) and b) end of high school (19 years). The difficulties identified in their study are properties of periodicity and the fact that trigonometric functions are not one-to-one. They further found out that students have a poor understanding of radian measures and a lack of its connection to the unit circle.

2.4 STUDENTS' UNDERSTANDING OF TRIGONOMETRIC FUNCTIONS

In looking at the students' understanding of trigonometric functions in the context of two college courses, the study explored the work of Weber (2005). The first course was taught by a professor unaffiliated with the study in a lecture-based course while the second course was taught using an experimental instruction paradigm based on Gray and Tall's (1994)'s notion of precept and current process-object theories of learning. This study used an interview and a paper and pencil test to collect data to get students' understanding of trigonometric functions for both classes. Students taught in the lecture-based course developed a very limited understanding of the functions while students who received the experimental instruction developed a deep understanding of trigonometric functions.

The researcher explored the study by Kepceoglu and Yavuz (2016), which investigated the effect of Geo Gebra in the teaching of the concept of the periodicity of trigonometric functions. They investigated how effective is the dynamic mathematics software Geo Gebra being used in the teaching of the periodicity of trigonometry functions, which is taught based on formulas in the context of traditional mathematics education. The study analysed and compared the effect of traditional teaching and computer-assisted mathematics teaching on students' conceptual learning about the periodicity of trigonometric functions.

Quasi-experimental was chosen as a design for the study and the workgroup was 36 Grade 10 high school students from a public high school in Insakral. Data were analysed using descriptive statistics and the results showed that the aid of computer mathematics education was more effective on students' learning than traditional mathematics education. Findings showed that cooperative learning was effective in teaching the periodicity of trigonometric functions.

In contrast, Brijlall and Niranja (2015), explored the role of manipulatives in the teaching and learning of trigonometric ratios in Grade 10. The case study design in the interpretive paradigm involved five Grade 10 pupils at a high school in South Africa. Observations, semi-structured interviews and activity containing written responses of pupils were used as data collection method in their study. The results from the study showed that the use of manipulatives in teaching and learning mathematics played a positive role in learners' understanding of trigonometric ratios at Grade 10 level. The results further showed that using manipulatives was an important mediating tool in the development of the conceptual and procedural understanding of mathematical concepts.

A study by Walsh, Fitzmaurice, & O'Donoghue (2017) explored the level of trigonometry Subject Matter Knowledge (SMK) of third- and final-year, pre-service second level mathematics teachers. The study determined if the sample of 50 teachers had sufficient level of SMK to teach second-level trigonometric concepts. It was reported that the sample of the forthcoming teachers had gaps in their SMK of trigonometric concepts that were on second-level syllabi and underprepared to teach trigonometric concepts for understanding. This study by Walsh et al. (2017) further claimed that the majority had enough SMK to teach early of Grade 9 trigonometry but not for later years.

In addition to the studies discussed above, Gur (2009) investigated the types of errors, underlying misconceptions and obstacles that occurred in trigonometry lessons. He used the sample of 140 Grade 10 high school pupils and six Grade 10 mathematics teachers. The study used a diagnostic test consisting of seven trigonometric questions - the students' responses were analysed and categorised. He found that most common errors that students made were improper use of the equation, the order of operations, and value and place of sin, cosine, misused data, misinterpreted language, logically invalid inference, and technical mechanical errors.

The researcher in this study also looked into Tuna & Kacar (2013) who investigated the effect of 5E learning cycle model, based on the constructivist approach, which is used for teaching trigonometry in Grade 10 of elementary mathematics education, on students' academic achievement and the performance of their trigonometry knowledge. Participants of the study are Grade 10 pupils registered for the spring semester of 2010-2011 at Anatolian High School in Kastamoun. The students were divided into two equal groups - (a) a control and (b) an experimental group. Students in the experimental group took the course about trigonometry from the researcher in an environment where the 5E learning models based on the constructivist approach were used. The students in the control group took the same course from their mathematics teacher in an environment where the activities of official mathematics were used. The performance of the experimental group was higher than the control group.

2.4.1 Intertwined strands of proficiency

Schools of thought such as Kilpatrick et al. (2001:116) developed five Strands of Mathematical Proficiency to capture the successful learning and understanding of mathematics, namely:

- Conceptual understanding – comprehension of mathematical concepts, operations, and relations.
- Procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- Strategic competence – ability to formulate, represent, and solve mathematical problems.
- Adaptive reasoning – capacity for logical thought, reflection, explanation, and justification.
- Productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These strands depend on each other; they are interwoven in the development of proficiency in mathematics (Kilpatrick et al., 2001).

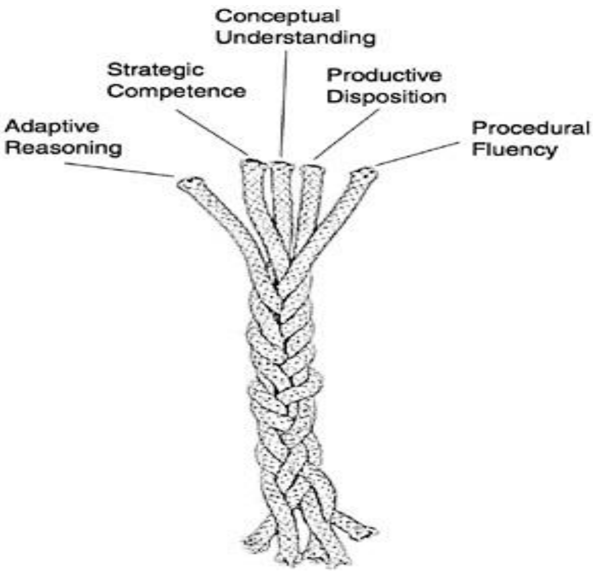


Figure 1. Intertwined strands of proficiency

This study adopted the conceptual understanding from the intertwined strands of mathematical proficiency by Kilpatrick et. al (2001). The study aimed to discuss mainly the comprehension of mathematical concepts, operations and relations. Conceptual understanding is one of five intertwined strands of mathematical proficiency chosen to capture what it means for anyone to learn mathematics successfully (Kilpatrick, *et al.,* 2001:115). Learners with conceptual understanding know more than isolated facts and methods because facts and methods learned with understanding are connected, easier to remember and use, and can be reconstructed when forgotten (Kilpatrick *et al.,* 2001).

The success indicators for conceptual understanding of trigonometry should be when students can respond correctly to trigonometric tasks that involve (a) general solution, and (b) basic ratio and reduction formulae since they demand higher-order thinking skill. The learners should be able to make connections of mathematical topics to solve trigonometric problems. For example, with general solution problems, learners should be able to correctly factorise (algebra) and apply basic ratios in connection with functions and graphs (pythagoras). The latter is supported by Kilpatrick, et. al when they say learners with conceptual understanding will know more than isolated facts and methods because facts and methods learned with understanding are connected and easy to reconstruct and remember.

2.5 COOPERATIVE LEARNING

Few studies conducted among students who were undertaking aspects of trigonometry found that there were additional findings that suggested that cooperative learning was instrumental to students' achievement. One study found that students who were enrolled in pre-algebra and used cooperative learning were not only capable of achieving higher than the control group, but they were more likely to remember the information that they were taught in the long-term (Duren & Cherrington, 1992). In addition, there is a body of knowledge on the effects of cooperative learning among middle school mathematics learners in 16 schools during the semester. The findings from this study are consistent with other results that show that cooperative learning has a positive effect on student achievement (Slavin & Karweit, 1984). The finding showed that learners who worked in cooperative groupings were significantly advantaged in terms of their scores as opposed to those in control groups who responded to the same tests. Additionally, Slavin & Karweit (1984) observed a higher level of scoring among middle school learners who were enrolled in general mathematics in cooperative learning groups compared to those who received the instruction of learning individually. Also, Sherman and Thomas (1986) observed similar findings in their study and reported that there were significantly higher improvements among middle school learners who took mathematics under the cooperative learning strategy compared to those who did mathematics as individuals.

The body of literature above shows the academic advantages that are attained through the employment of cooperative learning approaches in mathematics classrooms. In spite of this, the depth of the body of knowledge centred around achievement in mathematics at middle schools is shallow when compared to the number of studies at elementary grade levels.

The studies on the impact of cooperative learning have also pursued other variables besides that of achievement. The cooperative learning approach has been studied in the context of its correlation to other beneficial psychosocial or psychological results. One of these results is the quality of interpersonal relationships that develop during group work (Slavin & Karweit, 1984). Cooperative learning induces the necessary team experience that may, in the long run, be crucial to tackle learners' challenges in learning the intricate aspects of trigonometry, for instance (Augustine, Gruber, & Hanson, 1989).

Thus, cooperative learning instils the necessary social skills that are in any case relevant to the wider learning environment. Furthermore, aspects such as adolescence can interfere with students' proficiency in, otherwise, challenging courses such as trigonometry. Learners who struggle with trigonometry can begin to cope in ascendance by the interpersonal relationship opportunities that are presented in cooperative learning (Wood, 1987).

It is interesting to note that group work addresses this conflict provided the groups are small enough for individual recognition. Cooperative learning has also been linked to increases in self-esteem, attendance, time on task, enjoyment of school and classes, and motivation to learn as well as a decrease in dependence on the teacher (Augustine et al., 1989-90; Good, Reys, Grouws, & Mulryan, 1989-90; Slavin, 1990; Wood, 1987). Perhaps one of the most important benefits of cooperative learning has been more positive intergroup relations. Improved race relations, as well as increased acceptance of mainstreamed children, have frequently been reported (Augustine et al., 1989-90; Madden & Slavin, 1983, Slavin, Madden & Stevens, 1989).

Scholarly work has been able to identify the exposure to the social, interpersonal and psychological advantages of cooperative learning as crucial to learning trigonometry (Davidson, 1985; Mulryan, 1994). Complementary work by Mulryan (1994, 1995) found that junior learners who took trigonometry in their mathematics studies within the ambit of cooperating learning groups were exposed to more time with activities, fun in the classroom and were interested in investing more learning time within the entire group instruction. Learners and teachers in these trigonometry classes have reported that the advantages of learning trigonometry within cooperative approach settings were the acts of collaborating, intra-learner help and assistance, the interpersonal nature of it as well as the benefit of active participation (Mulryan, 1994).

Other schools of thought have, however, noted that there might be limitations to the usage of cooperative learning in trigonometry. Proponents of this view have argued that there might be interruptions in the learning process when group strategies are employed. Some scholarly writings have stated that learners participating in-group settings might succumb to complacency, especially the learners that are predisposed to the innate challenges of a course such as a trigonometry (Good, Reys, Grouws, & Mulryan, 1989-90; Mulryan, 1992-95).

From the literature available, limitations to cooperative learning are the time limits required for young learners to establish trust in groups and bonds with peers. This was a common drawback to students adapting to trigonometry as well as the accompanying learning materials (Good et al., 1989-90).

However, these drawbacks are comparatively benign to the results that have found significant benefits in learning trigonometry using cooperative learning and the psychological as well as social advantages. Some scholars tread between the lines such as Slavin and Karweit (1984), who argue that the capacity of students to respond to trigonometry instruction in a group setting, as well as the individual setup, are instrumental to the effective understanding of trigonometry.

A method known as Groups of Four, which employs neither group rewards nor individual accountability, has had few achievement benefits (Burns, 1981). Similarly, King (1993) observed third graders who were learning trigonometry in cooperative groups and found that high achievers dominated the work and decision making, while low achievers remained generally passive. The latter type of cooperative structure employed neither group goals nor individual accountability. Group rewards with individual accountability mean that each individual is rewarded only when others in his or her group also succeed, which prevents high achievers from dominating the work (Johnson & Johnson, 1987; Slavin, 1988). Thus, cooperative group work that incorporates both group rewards and individual accountability not only forces students to take responsibility for their mastery of the material but also makes the students responsible for their classmates' mastery (Slavin, 1990).

This study has taken an interest in one technique in cooperative group learning that consolidates the tenets of group reward as well as individual accountability, which is known as Student Teams-Achievement Divisions (STAD). The technique fosters competition among groups in addition to promoting teamwork in responding to trigonometry tasks (Slavin, 1990). The large membership of the group is required to learn trigonometry within the group setting because they only achieve higher scores when all the members contribute to solving the trigonometry activities. The team score depends on the performance of individuals in quizzes and when a team responds correctly to most of the trigonometry tasks, they accumulate rewards (Jaelani & Retnawati, 2016).

Specifically, individual points are awarded based on the percentage increase of a student's grade from one test to the next. Group points are calculated by adding all of the individual points together and dividing by the number in each group. In this type of reward structure, even the low achievers have the opportunity to contribute the maximum amount of points to the group. This type of cooperative learning structure has been well researched and consistently linked to significant gains in achievement and other favourable social or affective outcomes (Johnson & Johnson, 1981; Slavin, 1990).

2.6 THEORETICAL GROUNDINGS OF COOPERATIVE LEARNING

Cooperative learning is grounded in the belief that learning is most effective when learners are actively involved in sharing ideas and working cooperatively to complete academic tasks (Effandi & Zanaton, 2007). Furthermore, cooperative learning in a mathematics classroom involves social accountability, positive interdependence, individual accountability and groups' accountability (Kotsopoulos, 2010; Walmslay & Muniz, 2003).

Cooperative learning in mathematics instruction can also be described as the process with which students work together to accomplish a common goal under the guidance of their teacher (Bot & Eze, 2016). Felder & Brent (2001) further state that cooperative learning involves students working in teams or groups on problems or projects under conditions that assure both positive interdependence and individual accountability.

There are potential teaching-learning benefits when cooperative learning is used as outlined in the National Curriculum Statement (NCS) of 2012, CAPS Orientation (2013) Senior Phase Grade 7-9 Mathematics: (a) learners learn from each other, (b) improved mathematical expression, and (c) a broad range of mathematical content can be covered in one or two lesson(s). Jansen (2012) argues that depending on the structure used, cooperative learning can "promote conceptual understanding of mathematics development of mathematical reasoning skills in addition to procedural fluency." There are no studies in the literature that reported instances where cooperative learning is used in the context of trigonometry. Hence, this study worked around filling this gap.

2.7 IMPORTANCE OF COOPERATIVE LEARNING

Effandi (2003) investigated the effects of cooperative learning on students' achievement and problem-solving skills. His study of intact groups compares students' mathematics achievement and problem-solving skills. Cooperative learning method was used to teach experimental section while the traditional lecture method was used to teach problem-solving. The study found that students instructed with the cooperative learning method had a favourable response towards group work, and the use of cooperative learning method was a better alternative to the traditional instructional method.

Students' interaction and "talk" (Vygotsky's Constructivism) provides opportunities for students to think about and process the information. Time for "talking" and/or "writing" is needed to help students make sense of what they hear before attempting to "take in" more information. Cooperative learning promotes higher levels of achievement, greater depth of thought, improved attendance and encourages innovation in both teaching and student involvement.

2.8 ADVANTAGES OF COOPERATIVE LEARNING

The effects of cooperative learning on student achievement and attitudes in a secondary mathematics classroom were investigated by Whicker, Bol & Nunnary (1997). They used a quasi-experimental design to compare two pre-calculus courses; students in class 1 studied the material in cooperative learning groups while students in class 2 studied the material independently. Three-chapter test was used to measure student achievement, and a questionnaire was administered after the study to assess students' attitudes towards the cooperative learning procedure. Students in the cooperative learning group had increasingly higher test scores than students in the comparison group and significantly outscored the comparison group on the third chapter test. Most students indicated that they liked working in groups and appreciated getting help from one another, especially for learning difficult concepts.

Additionally, Tsay & Brady (2010) explored the relationship between cooperative learning and academic performance in higher education, specifically the field of communication. A questionnaire was used to administer students in a communication research course. The findings indicate that involvement in cooperative learning is a strong predictor of students' academic performance. Tsay and Brady (2010) found a significant positive relationship between the degree to which grades are important to a student and his or her active participation in cooperative learning.

Supplementary results Flynn (2013) examined the effects of cooperative learning on the academic achievement, classroom behaviour and attitude of students in mathematics. The effects of cooperative learning show the following: (a) a positive correlation between cooperative learning and increased test scores; (b) with the proper interventions, cooperative learning can help students learn how to be members of a cooperative learning community; (c) a cooperative learning environment can be beneficial to students who have previously had bad experiences with the subject; and (d) there are a variety of ways that cooperative learning can be used in the classroom. He further claims that cooperative leads to success in the general mathematics classroom.

This complements some of the views of scholars such as Slavin (2013) who found that well-structured methods such as cooperative learning produced more positive effect sizes than those evaluating, other instruction practices such as the use of innovative curriculum textbooks or the use of technology in reading and mathematics.

Similarly, Slavin, Lake, Hanley and Thurston (2014) stated that science teaching methods focused on enhancing teacher's classroom instruction throughout the year, such as cooperative learning and science – reading integration as well as approaches that give teachers technology tools to enhance instructions have significant potential to improve science learning. Interestingly, Slavin (2014) argued that cooperative learning as a pedagogical practice has had a profound effect on student learning and socialisation.

2.9 COOPERATIVE LEARNING IN A CLASSROOM SETUP

In looking at the issue of cooperative learning within the classroom environment, Slavin (1989) reported on a best-evidence synthesis of studies across both elementary and secondary schools that compared cooperative learning to control group studying the same material. The results showed that the overall effects of cooperative learning on achievement were positive in 72% of the comparisons whereas only 15% favoured control groups with 13% recording no significant differences. The study concluded that cooperative learning could be an effective strategy for increasing student achievement.

Moreover, Johnson & Johnson (2002) examined the effects of cooperative learning, competitive, and individualistic learning on several academic, personal and social dependent variable (i.e. achievable, interpersonal attraction, social support, self-esteem, perspective taking, learning together and controversy). The findings showed the strong effects sizes between cooperative learning compared to competitive and individualistic learning.

It was interesting to observe that studies by Slavin and Johnson & Johnson found that cooperative learning compared to competitive and individualistic learning had very strong effects on a range of dependent variables such as achievement, socialisation, motivation, and self-development.

Interest in cooperative learning gathered momentum in the early 1980s with the publication of the first meta-analysis involving 122 studies on the effects of cooperative, competitive, and individualistic goal structures on students' achievement and productivity in a sample of North American schools (Johnson, Marunyama, Johnson, Nelson & Skon, 1981).

The results showed that cooperative was more effective than the interpersonal competition and individualistic efforts. Interestingly, the results were consistent across all subject areas (language arts, reading, mathematics, science, social studies and physical education) for all age groups, and all tasks involving conceptual understanding, problem-solving, categorising and reasoning.

2.10 PRINCIPLES OF COOPERATIVE LEARNING

Getting students into groups and expecting that they would automatically cooperate and build beneficial relationships may prove unproductive. Students working in groups may find it hard to comprehend what is expected of them, and this may cause an eruption into discord among the group members who will inevitably face challenges in accomplishing the tasks at hand (Webb & Mastergeorge, 2003). The group members' inability to resolve their differences in opinions among themselves or controlling learners who will lag in the long run and are unable to withstand the rest of the learning process (Johnson & Johnson, 1990).

Scholars have posited five important principles that may underpin constructive positive and effective cooperative group learning situations (Johnson & Johnson, 1994).

2.10.1 Positive interdependence

It is the view of scholars such as Johnson and Johnson (1987) that positive interdependence is crucial to cooperative learning. Students must believe that their success depends on the success of their counterparts and that the only way they can achieve higher scores is when everybody succeeds. When undertaking trigonometry the principle of interdependence implies that all students must work together for a common goal; that is, to pass the activities. Positive interdependence can take the shape of asking group members to agree on a response to a trigonometry question or activity, then ensure that each person in the group can articulate the answer that is agreed upon and, thereafter, the group can tackle the assigned responsibilities (Johnson & Johnson, 1989).

Scholars such as Deutsch (1949) found that one of the success factors of cooperative learning was cohesiveness which develops in the group as a direct result of the perception of goal interdependence and interdependence among group members. These aspects are established in groups when students understand that they are each responsible for completing a part of the task which, in turn, all must achieve for the group to complete its goal. However, Jaelani and Retnawati (2016) say teachers can ensure that this occurs by assigning different parts of the group's task to different group members to complete.

2.10.2 Face-to-face Promotive Interaction

This aspect bears its prominence based on the existence of the first principle of positive interdependence. It requires trigonometry learners to deliberate the activity given, discuss their views concerning learning material while giving and receiving feedback and, most importantly, inspire each member of the group to complete their assigned activity (Webb & Mastergeorge, 2003). This may involve episodes of peer tutoring, temporary assistance, exchanges of information and material, challenging of each other's reasoning, feedback, and encouragement to keep one another highly motivated (Biehler & Snowman, 1997).

Successful cooperation is also incumbent on promotive interaction and the willingness of group members to encourage and facilitate each other's efforts to complete their tasks for the group to achieve its goal. Johnson and Johnson (1990) note that promotive interaction is characterised by students: providing each other with the help they need; sharing needed resources; providing effective feedback to group members on their performances on specific tasks; challenging other's conclusions and reasoning to promote clearer insights into the problem issue; and, working constructively together to attain mutual goals. Willingness to engage with others not only benefits recipients but also helpers as giving help encourages helpers to reorganise and restructure the information in their minds so they, in turn, develop clearer and more elaborate cognitive understandings than they held previously (Webb & Mastergeorge, 2003).

Teachers can facilitate interaction in groups when they ensure students sit near other group members, so they can hear what is being discussed, see each other's faces, and participate in the group's discussion. When students are provided with opportunities to interact with their peers during small group discussions, they learn to read each other's non-verbal language, respond to social cues, and engage in general banter about the work they are completing (Gillies, 2003). The third key component is individual accountability or one's responsibility in ensuring that he or she completes his or her share of the work while also ensuring that others complete theirs. The more students perceive they are linked, the more they feel personally responsible for contributing to the collective effort of the group. Johnson and Johnson (1990) maintain that teachers can establish individual accountability in two ways: firstly, by structuring positive interdependence among group members so they will feel responsible for facilitating others' efforts. Secondly, by holding students personally responsible for completing their part of the task and ensuring that their contributions can be clearly identified.

Assigning students to groups and expecting them to know how to cooperate does not ensure that this will happen. In fact, groups often implode because they lack the interpersonal skills required to manage disagreements among group members. These skills need to be explicitly negotiated (older students) or taught (younger children) and are the fourth key component in successful cooperative learning. In a series of studies that investigated the effects of structured and unstructured cooperative groups on students' behaviours and interactions, Gillies (2003) and Gillies and Ashman (1996) have consistently found that students who were trained to cooperate and help each other are more inclusive of others; respectful and considerate of others' contributions; and provide more detailed explanations to assist each other's learning than students who have not participated in this training. The social skills that facilitate students' interactions during small group discussions include:

- Actively listening to each other
- Sharing ideas and resources
- Commenting constructively on others' ideas
- Accepting responsibility for one's behaviours
- Making decisions democratically

2.10.3 Individual Accountability

Cooperative learning groups inspire each member involved to be stronger (Johnson and Johnson, 1987). Students in trigonometry classes learn how to function as a team so that they can accumulate the capacity to perform better as individuals when outside of the group setting. To counter the critics of cooperative learning, the principle of individual accountability ensures that students are not passive, and they are attentive to their portion of work.

Every individual's performance is evaluated and the feedback therein goes to the group or the individual alone. The group can utilise the feedback to determine who among the group may need more assistance in solving trigonometry activities, for instance (Johnson & Johnson, 1989). Teachers can prepare for individual accountability in cooperative groups by employing individual exams for learners or applying oral exams which would help monitoring group work in the process. Each of the participants in the trigonometry groups will have to be articulate when presenting the material learnt and comprehend the material that is being instructed throughout cooperative learning.

2.11 SUCCESS FACTORS IN COPERATIVE LEARNING IN TRIGONOMETRY

Certain enablers facilitate the success of cooperative learning in trigonometry. Johnson and Johnson (2009) believed that students in trigonometry needed to be taught the vital skills crucial for them to function in groups and the students must be inspired to apply those skills to grasp mathematical material as individuals and on behalf of or with others. It would not be farfetched that students achieve better in trigonometry when provided with feedback on how they could apply skills because feedback contributes to the betterment of their interpersonal relationships in the classroom as well as their understanding of trigonometry. One more success factor of applying cooperative learning in trigonometry is group processing. In this component, students have the opportunity to reflect on their progress as well as their working relationships. The teacher can apply the following questions when trying to usher in this type of reflection.

- What have we achieved?
- What do we still need to achieve?
- How might we do this?

Johnson, Johnson, Stanne, and Garibaldi (1990) assessed the impact of group processing on the success of 48 high school seniors and college students. The findings of their study showed that student experienced better performance gains when they participated in in-group processing discussion compared to those that were not exposed to these experiences. In Johnson, Johnson, Stanne, and Garibaldi (1990) said group processing involved making sure that all members of the group partook in summarising ideas and data, engaged in the discussions and cross-checked to see whether the group's work product was consistent with the interpretations of the work by all group members.

2.12 GROUP COMPOSITION AND TASK IN COOPERATING LEARNING IN TRIGONOMETRY

Against the background of the importance of cooperative groups in learning trigonometry that encompasses the five components highlighted above, the teacher additionally has to consider the make up of the group as well as its size. In a meta-analysis of 66 studies that examined the effects of within-class grouping (i.e. establishing small groups in classes) on student achievement in trigonometry at the elementary, secondary and post-secondary levels, Lou, Abrami, Spence, Poulsen, Chambers, & d'Apollonia (1996) found that students achieved higher outcomes when they worked in small cooperative groups than when they were not grouped, such occurs in traditional whole-class settings. Also, there were more desirable outcomes for students that worked in groups of three to four members in comparison to those that were in groups of more than five to seven members. This outcome possibly emanated from the fact that the latter design was akin to the whole class teaching set up wherein information is top-down as opposed to being constructed through participatory learning. Moreover, the impact of group capability organisation was different for students of different relative ability with low-ability trigonometry students learning more in heterogeneous or mixed ability groups while medium-ability students benefited significantly more in homogeneous groups.

Scholars such as Lou et. al (2001) found that composition has no impact with regards to high ability students who worked very well in a mixed or homogenous setting. Other results from the meta-analysis by Lou et al. (2001) found that small group learning in trigonometry was more effective compared to individual learning on students' achievement as well as group task performance. Group performance indicated better results in smaller groups of three to five members as compared to those who executed trigonometry tasks individually. In effect, students accumulated more individual knowledge when they were in a small group setting compared to those who worked individually.

In studies of different courses that used a theory-based meta-analysis of 123 studies that employed technology to support undergraduate student learning in distance education, Lou, Bernard and Abrami (2006) found that when media were used to support collaborative discussions among students in asynchronous distance education that is through discussion boards, email, the distance education students out-performed their peers who received classroom instruction only.

This finding is consistent with the findings of Lou et al. (2006) among trigonometry students which found that students involved in small group discussion had a higher chance of performing better compared to students who were not involved in discussions with their peers. Lou et al. proposed that the asynchronous discussions among students not only provided opportunities for elaborated feedback and help but these discussions may also have provided opportunities for students to learn reflectively and actively through peer modelling and mentoring. These recommendations for modelling and mentoring would retroactively assist students to develop higher metacognitive and self-regulated learning skills; skills which are strongly associated with successful learning in trigonometry.

2.12.1 Type of Trigonometry Task

The kind of trigonometry task that students have to tackle in class plays an instrumental role. The type of tasks has an impact on how the students interact within the cooperative learning framework in trigonometry (Cohen, 1994). The interaction within small groups as stated earlier is important to the overall achievement of the group. Shachar and Sharan (1994) argue that when learning trigonometry, there can only be effective interactions when the teachers are proactive in setting up the conditions that facilitate the functioning of students in small groups with regards to tasks that need the input of all group members.

This applies to ensure that teachers of trigonometry give students tasks that are open and discovery-based where all answers are valued. The only way for the students to successfully accomplish the task given is by effectively interacting with one another through the sharing and exchange of learning material.

Cooperative learning in trigonometry is such that input for the whole group is required to succeed as the learning material is not simply based on one individual’s knowledge.

According to Cohen (1994), when this occurs in trigonometry settings, it is the frequency of task-related interactions that are related to gains on computation and mathematical concepts and applications as well as on content-referenced tests in science with the most consistent predictor of achievement being giving detailed or elaborate information (Webb, 1991; Webb & Matergeorge, 2003).

In addition, Cohen (1994) argues that the significance of getting to an amalgamation of all the group members' contributions and the requirement that the group product will be presented to the wider class are structures designed to foster group cohesion and motivate students to complete the task. When teachers are proactive in structuring the small group activities such that these conditions are realised, students respond with better interaction, apply more techniques in solving trigonometry tasks, demonstrate a command of trigonometry when helping others in the group and can explain the problem they are faced with and how to effectively solve it.

2.13 BENEFITS OF COOPERATIVE LEARNING

Cooperative learning is widely recognised as a pedagogical practice that promotes socialisation and learning among students from pre-school through to tertiary level across different subject domains (Gillies, 2016). It involves working together to achieve common goals or complete group task-goals and the task they may not complete by themselves. Cooperative learning encourages students to discuss, debate, disagree and ultimately teach one another to master academic material.

It is the most successful approach incorporated with two key elements i.e. group goal and individual accountability to help students enhance academic achievement. Slavin further claims that in cooperative learning groups are rewarded based on the individual learning of all group members.

Group goals and individual accountability motivate students to give explanations and take one another's learning seriously, instead of simply giving answers (Slavin 1990). The success of one or more students helps others to succeed too (Slavin, 1987) and promotes the principles of shared leadership and responsibility (Penick & Alan, 1995). The above benefits confirm that cooperative learning is an important teaching and learning strategy.

Also, the benefits of group processing include enhancing of respect among group members from each other which, in turn, increase members' commitment to the groups, acceptance of group norms and contribute to an increase in members' collective identification (Johnson & Johnson, 2009). Providing students with feedback on how they use these skills not only helps to create more positive relationships among group members but also helps to increase students' achievement.

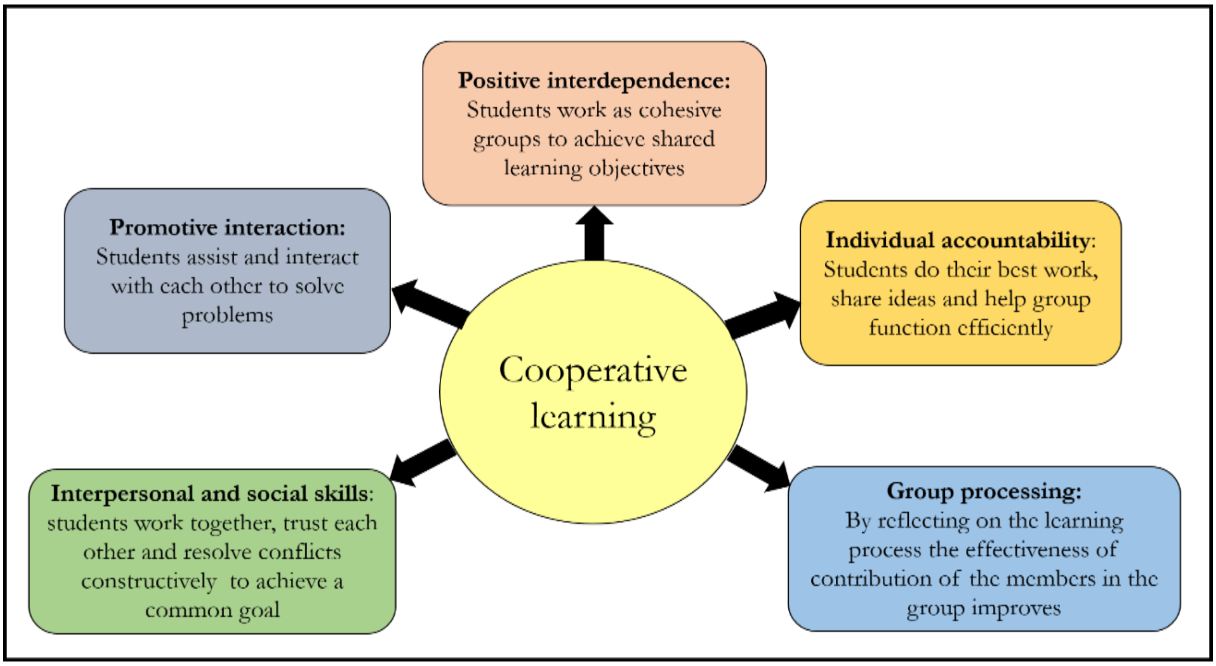


Figure 2. Benefits of Cooperative Learning (Johnson & Johnson, 2009).

2.13.1 Group learning

Interaction among group members is critically important to the success of small group activities and Shachar and Sharen (1994) argue that this will only happen when teachers create conditions that enable students to work in small groups on tasks that require cooperation among group members. In schools, opportunities for students to work in situations where they experience positive interdependence seems to be a better choice than situations based on negative or no independence.

Cooperative learning allows students to engage in discussion, take responsibility for their own learning and, thus, become critical thinkers. It assists students to do it by themselves as opposed to watching the teacher doing it and the students accepting every concept that is taught (Bot & Eze, 2016).

Secondary students are performing poorly in mathematics worldwide. Jailani and Retnawati (2016) pointed out some of the reasons that led to the underperformance of mathematics. The reasons included poor quality of teaching methods, techniques and strategies, negative attitudes, shortage of qualified mathematics teachers and poor learning environment. Bot & Eze (2016) then explains that the above problems can be minimised by a well-equipped learning and teaching strategy through proper on-the-job-training. There is a need for an investigation into the effectiveness of some of the innovative instructional methods, techniques and strategies particularly the application of cooperative learning in mathematics in secondary schools in South Africa.

2.14 PERCEIVED EFFECTS OF COOPERATIVE LEARNING IN TRIGONOMETRY

Some studies have examined the effects of cooperative learning method on students' learning achievement in trigonometry. These studies revealed that students taught using cooperative learning demonstrated better academic achievement. Ugwuadu and Abdullahi (2012)'s experimental research in Nigeria, using cooperative learning method, revealed a significant difference between the mean achievement scores of experimental and control groups in favour of the experimental group. A similar study by Muraya and Kimamo (2011) in Kenya also found that cooperative learning approach resulted significantly higher mean achievement scores in biology compared to the regular teaching method.

Moreover, Armstrong, Chang and Brickman (2007) compared cooperative learning approach and traditional lecture method and reported that the experimental group that was instructed through cooperative learning approach showed greater improvement in overall test scores than a control group that was taught using a traditional lecture approach. Their work further noted that the experimental group performed significantly better on questions requiring both factual knowledge and comprehension than students in the control group who were instructed through the regular lecture format.

The effect of cooperative learning has also been investigated in other subjects. Wachanga and Mwangi (2004) reported that secondary school students who were taught chemistry through the cooperative learning approach in Kenya's Nakuru district outperformed those who were taught through the traditional teaching approaches.

Similarly, Jaelani and Retnawati (2016) found that cooperative learning approach increased academic achievement and motivation to learn physics among secondary school students in Hong Kong as compared to those who were taught through the traditional teaching approaches.

However, a study by LaCarrubba (1993) using the STAD model of cooperative learning concluded that there was no significant difference in trigonometry achievement scores of primary school students taught through the cooperative learning method and those taught through direct reading activity, which was considered as a traditional teaching method. Similarly, More, Flowers and Abu (1992) compared the effects of STAD cooperative learning model with the traditional teaching approach in teaching economics and observed no significant differences in students' test scores between the control and experimental groups. It is evident from the foregoing that in most cases cooperative learning promotes students' academic achievements in trigonometry.

Such gains in students' academic achievement have been attributed to several characteristics of cooperative learning method. Ugwuadu and Abdullahi (2012) attribute cooperative learning method to facilitating better conceptual understanding of the subject matter as students share ideas and point of views, give and receive support from each member who helps to dig below the superficial level of understanding of the material they learn. Humphrey, Johnson and Johnson (1982) attribute academic achievement to the support students receive from their group members.

They explained that individual students tend to give up when they get stuck, whereas students in cooperative learning groups find ways to keep going, which leaves more chances to be successful academically. Many studies have also investigated the effect of cooperative learning method on students' attitude towards various subjects. Student (2015) examined the effect of cooperative learning method on students' achievement and attitude towards biology in secondary schools in Delta State in the United States of America and concluded that cooperative learning method increased students' achievement and promoted positive attitude towards studying trigonometry.

A comparative study by Humphreys, Johnson and Johnson (1982) found that students studying physical science in a cooperative learning treatment group rated their learning experience more positively than students in competitive and individualistic treatment groups. In a study involving elementary and secondary students who were taught nutrition, Wodarski, Adelson, Todd and Wodarski (1980) found that 95% of the elementary students enjoyed the cooperative learning activities and had learned a lot about nutrition. However, some studies found that cooperative learning does not affect students' attitude. For example, Bialangi, Zubaidah, Amin and Gofur (2015) found no difference between the attitudes of students taught by jigsaw and guided inquiry in Natural Science class in senior high school in Palu, Central Sulawesi, Indonesia.

Schickler (1998) also found no significant differences in post-test attitudes toward trigonometry knowledge between the EG and CG in an urban community college in Florida, USA. Literature is abundant on the effect of cooperative learning method on students' learning achievement and attitude. However, the literature review has not revealed any studies on the effect of cooperative learning approach on students' trigonometry achievement scores and attitude towards the subject in Bhutan. Therefore, this study attempted to fill this knowledge gap and contribute to the body of knowledge on the cooperative learning approach.

2.15 EXPERIENCES OF LEARNERS USING COOPERATIVE LEARNING

The problem-solving skill has a central position in trigonometry. The abilities to solve trigonometrical problems are a higher-order thinking skill. According to Polya (1973), there are four steps of trigonometrical problem-solving: understand the problem, devise a plan, carry out the plan, and look back (Yuan, 2013). Trigonometrical problem-solving is a process which helps the students learn the concept and skill (Tambychik & Meerah, 2010). However, teaching problem-solving in trigonometry using cooperative learning is not easy at times.

In light of that, Demitra and Sarjoko (2017) have implemented the model of IMPROVE (Introducing new concept, Metacognition questioning strategy, Practicing, Reviewing and reducing, Obtaining mastery, Verification, and Enrichment) in a cooperative learning setting and found that the Grade 10 students of the senior high school to counter some of the challenges that might emerge. The IMPROVE model can develop rational exponent understanding of students' rational exponent concept partially. Nevertheless, students within the cooperative learning set up may not be able to make a relationship between prior knowledge in solving the rational exponent problem.

In Indonesia, students had difficulty when they learned trigonometry problem-solving in the cooperative learning environment. According to classroom observation results of Demitra and Sarjoko (2017), some students reported difficulty in cooperating with other group members. The steps of cooperative learning were reported to be unclear for students. Teachers argued that the process had not been optimal, leading to less successful mathematical problem-solving process in trigonometry. Work by Jaelani and Retnawati (2016) found that teachers had difficulties in implementing cooperative learning. Other countries such as Canada and Spain in Sharif and Gisbert (2015) showed the cultural impact on the teacher to design effective cooperative learning strategies. Furthermore, Sharif and Gisbert (2015) argue that the problem of developing problem-solving skill related to rational exponent can be solved by developing a model of cooperative learning that considers the cultural background of the student. The importance of cultural background like indigenous knowledge has widely been recognised in learning trigonometry (Verner et al., 2013; Weldeana, 2014).

The study of Bringula et al. (2016) show that depending on prior knowledge, students can understand more of the terms of equations and the following steps in solving equations. Hohensee (2016) said that the performance of the problem solver is to mobilise the relevant elements of his knowledge and connect them with the element of the problem.

Central to the goals of cooperative learning in trigonometry is the enhancement of achievement, problem-solving skills, attitudes and inculcate values. Effandi (2003) investigated how cooperative learning affected student achievement and problem-solving skills. This study of intact groups compares students' trigonometry achievement and problem-solving skills. The experimental section was instructed using cooperative learning methods and the control section was instructed using the traditional lecture method. Cooperative group instruction showed significantly better results in trigonometry achievement and problem-solving skills. The effect size was moderate and therefore practically meaningful. The study by Effandi (2003) also found that students in the cooperative learning group had a favourable response towards group work. He concluded that using cooperative learning methods was a preferable alternative to the traditional instructional method.

The study by Eam (1999), using teams games tournament (TGT) and STAD as a model, found that students who were taught with a cooperative structure outperformed the students in individualistic goal structure in mathematics problem-solving. Other researchers have reported similar findings that point to the achievement benefits of using cooperative learning (Demitra & Sarjoko, 2017).

2.16 THEORETICAL FRAMEWORK

Constructivism was chosen as the theoretical framework for this study because it emphasises the active role of students in building and making sense of knowledge. Constructivism approach applies to cooperative learning in enhancing conceptual understanding of trigonometry. Constructivism is an epistemology which claims that humans construct knowledge and meaning from their own experiences (Alam, 2017). Constructivists believe that learning should take place in realistic and authentic settings and that knowledge is constructed from experience.

For Idris and Chan (2017), the constructivist method of instruction recognises the importance of the learner in the learning process and believes that learners should build their own understanding of concepts through self-discovery. Therefore, knowledge is not obtained passively but is actively and continuously adapted by structuring and re-structuring information and experiences as the learner develops to a higher level of understanding (Idris & Chan, 2017).

In addition, constructivism is a view that emphasises the active role of students in building and making sense of knowledge (Kang & Han, 2017). Constructivism has two views: a psychological view and social view. This research followed the Vygotsky's social view because learners will be interacting with one another and share their views. Thus, generate a shared understanding related to the concept (trigonometry) to make sense of their own experiences (Idris & Chan, 2017).

Furthermore, Vygotsky (1986) argues that human activities take place in social settings and cannot be understood apart from these settings (Gredler, 2012). One of his key ideas was that our specific mental structures and processes could be traced to our interactions with others. These social interactions are more than simple influences on cognitive development, they are actually creating our cognitive structures and thinking processes (Palincsar, 1998). Regarding higher mental processes, Vygotsky believes that an individual can direct their own attention and thinking through problems. This is first co-constructed during shared activities between the individual and another person. Thereafter, these co-constructed processes are internalised by the individual and become part of their cognition (Gredler, 2012). In essence, for Vygotsky, social interaction was more than just influence but the origin of higher mental processes. Vygotsky said cultures were actually formed through tools and symbols. Intelligence is achieved when a learner can “internalise” the tools that are being provided in the culture itself. When the tools of a culture evolve and emerge, the learners’ ability to grow as individuals and increase their knowledge base is broadened (Gredler, 2012).

Additional cognitive skills that develop among children are the ability to distinguish fantasy from reality, to describe similarities between two objects, and to apply creative thinking to problem-solving (Feldman, 2016). When people learn, the human capacity for information processing is limited. According to Von Wright (2000), automation of activities and thinking strategies is a means of relieving the cognitive load as complex activities and tasks demand a lot of concentration and cognitive capacity at first, thereafter, in the long run, easier constituent functions become automated. Scholarly work by Goodman and Hammill (2018) found that such allowed human beings to focus their attention more selectively. It is thus imperative that cognitive skills are developed and enhanced early in life (Bloom et al., 1956; Anderson et al., 2000).

The success indicators for conceptual understanding of trigonometry should be when students can respond correctly to trigonometric tasks that involve: (a) reduction formulae and (b) basic ratio since they demand higher-order thinking skill. The learners should be able to make connections of mathematical topics to solve trigonometric problems. Using reduction formulae, they should also be able to correctly factorise (algebra) and understand (then apply) basic ratios in connection with functions and graphs pythagoras. The latter is supported by Kilpatrick et al. (2001) when they said learners with conceptual understanding knew more than isolated facts and methods because facts and methods learned with understanding were connected and easy to reconstruct and remember.

2.17 CONCLUSION

This chapter discussed the reviewed literature on the conceptual understanding of trigonometry and the history of cooperative learning as well as the tenets or principles that underpin cooperative learning and the vital success factors to cooperative learning in trigonometry. The literature on the perceived benefits of cooperative learning was deliberated and a critical highlight of the perceived impact of cooperative learning was outlined. This section also reviewed some literature on the experiences of learners who have been exposed to cooperative learning and some essential feedback from teachers was incorporated. The next section discusses the methodology used to gather data for this study.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

This section provides the methods used to collect data to answer this study's research questions. According to Creswell and Creswell (2017), identifying the research methodology is important because it ensures that a study can be undertaken in an organised manner. The elements of research methodology that are discussed include research questions, research methodology, the adopted design, the research approach, the study site, the population of the study, the sample size and the techniques, instruments, data analysis as well as the considerations that were undertaken concerning ethics.

3.2 RESEARCH QUESTIONS

The research questions that guided the study are:

- What are teachers' views of cooperative learning in the classroom?
- What are the cooperative learning opportunities for students in trigonometry?
- What is the effect of cooperative learning on students?
- What cooperative learning strategies can help improve students' competence in trigonometry?

3.3 RESEARCH METHODOLOGY

Scholars describe research methodology as a set of logical and systematic procedures which are to be followed in research to gather the appropriate data for the study (Kothari, 2012). The research methodology therefore validates and justifies how the data was obtained. There are two research methodologies, namely quantitative and qualitative methodologies (Naoum, 2012), and this study adopted the qualitative approach. These two approaches are thoroughly discussed below on section 3.5 about research approach. Qualitative research is an approach for discovering and understanding the meaning individuals or groups attribute to a social problem (Creswell, 2014). It involves emerging questions and procedures. Data in this approach is collected in the participants' setting (Creswell, 2014).

The qualitative methodology was selected to enhance the conceptual understanding of trigonometry when cooperative learning is used. Furthermore, to understand the meanings constructed by students, how they make sense of their own experiences and the knowledge they have about trigonometry (Merriam, 2009:13).

3.4 RESEARCH DESIGN

A research design describes the actionable part of the research which is crucial to connecting the research questions to the eventual implementation of a study (Creswell & Clark, 2017). This design is, therefore, the guideline from which a study can be developed and concluded (Sekaran & Bougie, 2016). It is a basic plan that guides the data collection and analysis processes of conducting research. There are different types of research designs. They are case study, exploratory and causal research design. However, this study used the single case study as a research design because scholars such as McMillan (2012) argue that it offers an analysis of one or more events, settings, programs, social groups, or individuals in their natural setting. Moreover, the research design was chosen because case studies are time-restricted, and researchers collect detailed information using a variety of data collection techniques over a sustainable period (Yin, 2012). There is a dearth in information around the research problem of this study. As such, the single case study research design was more appropriate for this study. According to Walliman (2017), a case study is conducted when the researcher wishes to understand new knowledge that could bear on the status quo and other norms. In this case, the researcher conducted a case study to ensure that the research questions were framed around the research problem. The researcher in this study collected data through semi-structured interviews, observation and documents for three weeks. The case of Grade 11 mathematical classroom was used to collect data and to analyse students' understanding of trigonometry when cooperative learning was used.

3.5 RESEARCH APPROACH

Below is a discussion of the two paradigms as well as an explanation of how they influence the research being conducted and what makes them distinct from each other.

3.5.1 QUANTITATIVE RESEARCH APPROACH

According to Walliman (2017), quantitative research refers to studies that seek to measure some concepts or phenomenon of interest using variables, hypotheses and unit of analysis. Quantitative studies are aimed at describing novice ideas, situations or concepts (Sekaran & Bougie, 2016). It is grounded in the positivist philosophy that views reality as objective, observable, value-free, inert and stable. According to Creswell and Clark (2017), quantitative research approach begins with research questions or hypotheses and thereafter design an approach that is appropriate for providing the numerical bases for confirming or refuting the hypotheses or to answer the research questions. Quantitative research applies deductive reasoning methods to argue from what is unknown to reach a logically certain conclusion. This method was not used in this study because the study sought an in-depth understanding of cooperative learning and numerical description of the phenomena was deemed insufficient. A qualitative approach was used instead.

3.5.2 QUALITATIVE RESEARCH APPROACH

Qualitative research was used in this study. The qualitative research approach refers to a method of study that displays, analyses, summarises and interprets words and images based on the raw data that was collected by a researcher (Brace, 2018). Walliman (2017) describes qualitative research as “studies that are subjective, but in-depth, using a probing, open-ended and a free-response format.” Qualitative research was chosen for this study because it allowed the researcher to gather data that is rich in depictions of how school students experienced the phenomenon of cooperative learning as a way of learning trigonometry (Noaum, 2012). The approach helped the researcher understand the meanings constructed by students; how students make sense of their own experiences and knowledge they have about trigonometry (Merriam, 2009:13).

3.6 RESEARCH PHILOSOPHY

Research philosophy can be described as the views about how empirical data can be gathered, analysed, interpreted and utilised (Saunders, Lewis, & Thornhill, 2016). According to Saunders, Lewis and Thornhill (2016), research philosophy can be described in terms of the ontology and the epistemology. Ontology centres more on the nature of reality whereas epistemology deals with what is regarded as truth (Saunders, Lewis, & Thornhill, 2016). The philosophical assumptions are important because they determine the method used to collect data as well as the approach for a study.

In this study, the interpretive paradigm was employed to understand the experiences of students under the use of cooperative learning in a trigonometry classroom. Creswell and Creswell (2017) are of the view that the interpretive paradigm is appropriate for such studies because in this case, it helps with the understanding of the perceptions of students through their experiences within a cooperative learning environment. Therefore, this study used the responses of students to cooperative learning to construct and interpret their understanding from the collected data (Cao Thanh & Le Thanh, 2015).

3.7 POPULATION AND SAMPLING TECHNIQUES

The target population is described as a group of people that share the same characteristics with which the researcher can use for the purposes of making generalisations to the general population (Welman, Kruger, & Mitchell, 2005).

3.7.1 Research site

The population of this study was thirty-one (31) Grade 11 students and their teacher at a school in Limpopo Province, Capricorn district, Moletlane Circuit. Matladi Project High School was purposely selected mainly because of its diversity. This rural school is located at Moletlane village, Zebediela in Limpopo province. The school has an average enrolment of about 900 students from Grade 8 – 12, and about 55% of students in the Further Education and Training (FET) phase (Grade 10-12) are doing mathematics.

The mathematics classes in this school have 30 students on average per class and each grade has two mathematics enrolled classes. Matladi is a government non-paying tuition fee school. The school is well resourced with three (3) computer laboratories, two (2) science laboratories, and one (1) library. The school is considered as one of the best schools in the circuit as it has a large number of student enrolment from various villages around the school. Also, the school is always in the top 5 best performing schools in terms of matric results at the circuit level. One Grade 11 mathematics classroom was selected at this school for data collection purposes.

TABLE 3.1: THE TEACHER'S DEMOGRAPHIC DATA

Characteristic	Teacher S1
Gender of teacher	Male
Professional qualification	Diploma in Mathematics Education
Subject major	Mathematics
Mathematics teaching experience	24 years

TABLE 3.2: THE STUDENTS' INFORMATION

Gender		
Male	Female	Total
13	17	30
Age range		
15-17	18-20	Total
24	06	30

Table 3.3: Summary of selection criteria of the participants

Participant	Instrument used	Selection criteria
Teacher S1	- Interview - Lesson observation	- Purposive sampling - Purposive sampling
Student L1 –L30	- Interview - Lesson observation	- Purposive sampling - Purposive sampling

3.8 SAMPLING

Sampling involves choosing the appropriate sample that can represent the entire population during a study (Sekaran & Bougie, 2013). Probability sampling strategy and the non-probability sampling strategy form the main types of sampling (Sekaran & Bougie, 2013). According to Van Zyl (2014), a probability sampling strategy is one where every item in the population has a chance of being selected whereas, in non-probability sampling, some elements of the target population have a higher chance of being chosen. In this research, a probability sampling method was selected. The sampling was purposive as the researcher selected the school that had many mathematics students and the school was close to the researcher’s home. Purposive sampling ensured that respondents who could answer the research questions were selected to participate in the study.

The criteria for the selection of the Grade 11 teacher was his experience of teaching mathematics at that level or FET phase, while the sample of participants (students) was selected based on the nature of the study i.e. Grade 11 mathematics classroom the selected teacher was responsible for teaching.

3.9 DATA COLLECTION TOOLS

The instrument that is used to gather data is referred to as the data collection tool (Van Zyl, 2014). The collection of data in this study was through: a document (groups’ written task), structured interview and lesson class observation.

3.9.1 Interviews

An interview is a two-way conversation where the interviewer asks questions to learn about the ideas, beliefs, views, opinions, and behaviours of the participants. Interviews are the most widely used method of data collection, they are in-depth and either semi-structured or unstructured (McMillan, 2012: 291). Semi-structured interviews were used in this study because they allowed the researcher to have an interactive dialogue with research participants. The semi-structured interviews also allowed the researcher to ask follow up questions and better understand the responses of participants instead of taking responses at face value. The same interview questions were used to get their views, opinions and beliefs of learning in small groups than in the traditional textbook method. Qualitative interviews allowed the researcher to see the world through the eye of the participants as the participants' responses to the questions gave an in-depth knowledge to the study. The interview guide (questions) is an important aid that keeps the researcher with needed consistency (Kranss, Hamzah & Nor, 2009). Interview questions were used for students and a teacher. Grade 11 teacher and students were interviewed to get more understanding of how they felt and thought about the use of cooperative learning in a mathematics classroom.

3.9.2 Interview guide design

The questions in the interview guide expanded on the four research questions highlighted in Chapter 1 to form seven questions for the students and a separate list of six questions for teachers. The questions were open-ended and each question was allowed a 3-minute response. While the questions were open-ended, the researcher ensured that the interviewees kept time to provide in-depth information that was only relevant to the questions posed. The researcher, however, gave room for some more time where a research question was not answered. The guide avoided leading questions. The questions were posed using simple language and were kept concise. For example: *"What are your difficulties in learning and understanding trigonometry?"* and *"How does cooperative learning enhance your understanding of trigonometry?"*. The teachers was asked questions such as *"What are the benefits of learning* *mathematics using cooperative learning?"* and *"What is your preferred method of teaching and why?"*

3.9.3 Observation

The researcher was constructivist as mention in Chapter 2 (2.16). In this research I carried the idea that learning should take place in realistic and authentic settings and that knowledge is constructed from experience. Observations were used for recording the behavioural patterns of teachers and students (McMillan, 2012). Students' responses and communication with one another and with that of an educator indicated the behaviour patterns and communications. The observation guide was used as a guideline to record the behavioural patterns of students during the lesson delivery of trigonometry in a Grade 11 classroom. McMillan (2012) states that observations should be recorded as field notes (the detailed written descriptions of what was observed, as well as the researcher's interpretation).

The learners presented to the class how they resolved the tasks given in class. In addition, each of the group members contributed to the final presentation and convinced the teacher that they understood how the group came to the answers that were revealed to the rest of the class. As a constructivist, I was conscious of the importance of the learner in the learning process and believes that learners should build their own understanding of concepts through self-discovery. This ensured that knowledge is not obtained passively but is actively and continuously adapted by structuring and re-structuring information and experiences as the learner develops to a higher level of understanding (Idris & Chan, 2017).

For instance, in lesson 1 that observed for an hour, the teacher placed learners in groups of 6, and there were 5 groups in the classroom. The learners were introduced to the topic using revision activity of the three trigonometric ratios (sin θ, cos θ, tan θ, cot θ, sec θ, and cosec θ) learnt in Grade 10. After 20 minutes of teaching the ratios, the teacher instructed the learners to work in groups on a problem he gave them to resolve and thereafter present the solutions to the class. Group 1, 2 and 5 among the five, participated actively in this revision lesson.

3.9.4 Observation guide design

The active role of students in building and making sense of knowledge was followed in the design of the observation guide (Kang & Han, 2017). Constructivism has two views: a psychological view and social view. The observation guide design followed the Vygotsky's social view because learners will be interacting with one another and share their views. Thus, generate a shared understanding related to the concept (trigonometry) to make sense of their own experiences (Idris & Chan, 2017).

The observation guide included observations of teaching and learning approach of cooperative learning; the essence of trigonometry case in point basic ratios and reduction formula; and the strength of trigonometry in terms of application and problem-solving. The observation guide also intended to observe the effectiveness in addressing or responding to trigonometric questions or problems; promoting students' active engagement and promoting interaction among students. The observation guide comprised a Likert scale of 1 to 4 with 1 being the lowest and 4 being the highest. This scale was used to rate the performance of the teacher and the students in the classroom. Each rating included a concluding remark justifying it. The interaction between the teacher and students was observed and relevant details were documented when learning and teaching of trigonometry took place every day for three weeks.

3.10 DATA ANALYSIS

Qualitative data analysis is the classification and interpretation of visual or linguistic material to make statements about implicit and explicit dimensions and structures of meaning-making in the material and what it represents (Flick, 2013).

3.10.1 Analysis of Interviews

The interview data were analysed using thematic analysis. Braun & Clarker (2006) define thematic analysis as the process of identifying patterns or themes within qualitative data. The researcher organised collected data by searching for patterns and themes collected and then transcribed data recorded during interviews (Flick, 2013).

This approach enabled the researcher to report the experiences of the participants gathered during the data collection process. The study used thematic analysis to find common patterns across a data set. To analyse the data, recordings of interview responses were read by the researcher thoroughly. This consisted of the first stage of analysis referred to as familiarising with the data. The researcher then collated the data according to the research questions they answered. For instance, information about the benefits of cooperative was combined as well as information about teachers' preferred methods for teaching trigonometry. After that, the researcher reviewed the themes to ensure that they provided the information that answered the research questions. The review stage also helped to ensure that data fell under the appropriate theme. This was important for the next stage of thematic analysis which was naming or labelling the themes. Relevant phrases or sentences were labelled to search for themes with broader patterns of meaning, review the themes to make sure that they fitted in the data. The researcher labelled the categories and decided on their relevancy and how they were connected. The categories and connections were the main results of the study. The researcher identified and consolidate comments about each trigonometric topic that is said by the participants about each posed question of trigonometry during the interview, discuss and draw conclusions.

3.10.2 Analysis of Observations

The researcher read through the observation reports step by step which rated the extent to which teaching and learning approach of cooperative learning was executed in the classroom; the essence of trigonometry such as basic ratios and reduction formula as well as application and problem-solving in trigonometry. With constructivist guidance, the researcher sought themes informed by learner engagement with trigonometry learning material as well as active participation with peers and the teacher in the classroom. Constructivism was implemented through analysing and rating the teacher's responses in addressing to trigonometric questions or problems;

promoting students' active engagement and promoting interaction among students. The behaviour of the students and the teacher in the classroom was analysed through a table that used a scale of 1 to 4 with 1 being the lowest and 4 being the highest. This scale was used to rate the performance of the teacher and the students in the classroom. Each rating included a concluding remark justifying how the researcher rationalised their rating.

3.11 PILOT

Pilot study is a preliminary small-scale study that researchers conduct to help them decide how best to conduct a large research project (Creswell, 2013).

3.11.1 Piloting the Interview Guide

Piloting for an interview is an integral aspect and useful in the process of conducting qualitative research as it highlights the improvisation to the major study (Majid & Mohamad, 2017). The Grade 11 teacher and ten (10) Grade 11 students were the participants during the piloting of interviews. The research questions from the instrument were used with the participants and their response to the interview questions helped refine the research questions and the adjustment of the interview guide. The teacher's response also helped in reformulating questions to avoid ambiguity and indirect repetition of questions from the interview guide.

It helped the researcher to refine the research questions and finalise the methods that were best to pursue the study. Instead of asking participants what they understood about cooperative learning, for example, the researcher ended up opening the interviews by asking about group work, which was the best way with which students understood cooperative learning. The pilot study also helped the researcher to estimate the time and resources that were necessary to complete the larger study (Collingridge, 2016). The open-ended questions were estimated to consume five minutes per question, which was perceived to be lengthier. Therefore, the pilot study helped the researcher to anticipate any diversion by participants during the actual data collection process.

3.11.2 Piloting Observation instrument

The researcher observed the Grade 11 teacher and Grade 11 mathematics students during the presenting of a lesson on trigonometry and then used the observation guide to see the enhancement of conceptual understanding of trigonometry when cooperative pedagogy was used. There were ten bases for observing the interaction between the teacher and the students in the classroom and after the piloting they were reduced to seven. This is because observations were condensed into the broader issue about interaction, for instance. The teacher and the students were observed during the period of teaching and learning trigonometry in a classroom, and that helped in reviewing the observation schedule so that it could answer the research questions.

3.12 CREDIBILITY AND TRUSTWORTHINESS

Credibility and trustworthiness intended to have students, external reviewers or the data sources themselves provide evidence of the accuracy of data collected in the study (Creswell, 2014). Mcmillan (2012) further emphasise that credibility is the primary criteria for evaluating qualitative studies. The application of this is explained below.

3.12.1 Credibility

Mcmillan (2012) defines credibility as the extent to which the data collected, data analysis and conclusions are accurate and trustworthy. To make the data credible, the pilot studies were used to develop questions which encouraged full discussion of the research problem and an analysis of the results. From the pilot study, the researcher noted that some questions diverted the attention of respondents from cooperative learning into an in-depth description of trigonometry and maths. The researcher carried out the final data collection with this in mind and was proactive in making sure that cooperative learning was the core issue for interviews. Interviews also ensured the collection of in-depth data. This together with the pilot study, therefore, ensured that the results were kept in the context of cooperative learning and used only to answer the questions asked.

3.12.2 Triangulation

Triangulation is the method of using more than one method to collect data on the same topic to assure validity. It is a technique that seeks convergence of findings and cross-validation among methods of data collection (Mcmillan, 2012). This study used more than one method (observations and interviews) to collect data to validate the collected data. Interviews helped in narrating the experiences of the students and the teacher whereas the observations rated the degree to which those narratives were practised in the actual classroom. So, the participants' responses were not just taken at face value but tested in real-life situations.

3.13 ETHICAL CONSIDERATIONS

Ethical issues are about negotiating how to get access to the people and sites being studied, how long to stay in the field and the way of collecting data - how to interact with participants respectfully (Ryen, 2009). The researcher explained the purpose of the study to all Grade 11 mathematics students and their teacher.

3.13.1 Permission

Permission to research at Matladi Project High School, stating the purpose and the significance of the study, was sought through a letter to the Capricorn District Director, the Circuit manager and the school principal. During the first visit to the school, the researcher talked to the students and their teacher about the study, informing them that the purpose of the study was only for educational purposes and information obtained would not be used to tarnish their reputation.

The parents of all the minors were written a consent letter to allow their children to participate in the study. The letter explained that the students would take part in an interview. There was no foreseeable risk to the child as the participant. It was explained that there would not be any remuneration for participating in the study. The teacher and students' participation was voluntary and they had the freedom to decline or withdraw from participating at any time.

3.13.2 Confidentiality

The analysis of the data as an ethical consideration was included to protect the participants' identity and confidentiality. This study used pseudonyms for the school and the participants for their privacy and anonymity. Collected data was stored in a safely locked cupboard and electronic data on a password-protected computer at the researcher's home for five years for future research or academic purposes.

3.14 CONCLUSION

This section outlined the research methodology of the study and the various elements of the research process, such as research design, research approach, data collection instrument and data analysis. This study adopted a qualitative research approach and collected data were analysed using thematic analysis. The next chapter presents and discusses the results of the study.

CHAPTER 4

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

This chapter presents the analysis and discussion of findings of the study that investigated the use of cooperative learning to enhance conceptual understanding of trigonometry in a Grade 11 mathematics classroom. Qualitative interviews were used to allow the researcher to see the world through the eyes of the participants as they responded to the questions to give in-depth knowledge. The lesson observations served to triangulate the data.

The findings are analysed using the literature noting whether these are consistent with or divergent from the existing body of knowledge. Findings were presented under each research question posed in Chapter 1 as follows:

- What are teachers' views of cooperative learning in the classroom?
- What are the cooperative learning opportunities for students in trigonometry?
- What is the effect of cooperative learning on students?
- What cooperative learning strategies can help improve students' competence in trigonometry?

Pseudonyms are used in respect to the confidentiality of the respondents - S1, S2, S3, S4, S5, S6, S7, S8, S9, S10 and S11.

4.2 RESEARCH QUESTION 1

What are teachers' views of cooperative learning in the classroom?

This part of the data presentation and analysis highlights and analyses the findings from the teacher about his views of cooperative learning as he uses it in the classroom. In this section, data is presented, analysed and discussed under the ambit of cooperative learning in teaching trigonometry. Data to answer Research Question 1 was gathered through a semi–structured interview and observations.

4.1.2 Cooperative learning in teaching trigonometry

The teacher provided his views of cooperative learning. The account from the teacher was necessary because it helped the study to gather his perception of the subject matter before undertaking the observations that are discussed in detail in section 4.4 of this chapter. The teacher said:

To me, cooperative learning is when the learners have equal opportunities to work on problems and exercises and then learn from each other on how trigonometry problems are solved.

He added:

I think cooperative learning is a way to make learners participate in the classroom. It helps when they talk to each other about what I am teaching them and how they understand it.

The teacher also spoke about the dynamics of cooperative in his classroom:

Cooperating learning for me describes the activities that my learners do in groups. It describes the activities that I give them to work out with each other and present to each other, me and the rest of the classroom.

The teacher said that they believed that learners learnt best from each other and using group work.

What I do is give my learners a set of problems to respond among themselves. I place them in groups of six so that they are enough to have a variety of contributions but also not too big to the extent that the other learners' contributions are overshadowed. The desks face each other which creates face-to-face cooperation when working together.

The teacher also described further their applications of cooperative learning in the classroom:

I often give them ten minutes to work out the trigonometry problems and, afterwards, each of them is supposed to explain.

The teacher's view of cooperative learning involved giving equal opportunities to work on problems and exercises. The teacher believed that these opportunities helped the learners learn from each other to solve trigonometry problems.

From the observation, the teacher implemented this understanding in practice. After the ten minutes lapsed, the learners presented to the class how they resolved the tasks given in class. In addition, each of the group members contributed to the final presentation and convinced the teacher that they understood how the group came to the answers that were revealed to the rest of the class.

For instance, in lesson 1 that observed for an hour, the teacher placed learners in groups of 6, and there were 5 groups in the classroom. The learners were introduced to the topic using revision activity of the three trigonometric ratios (sin θ, cos θ, tan θ, cot θ, sec θ, and cosec θ) learnt in Grade 10. After 20 minutes of teaching the ratios, the teacher instructed the learners to work in groups on a problem (see revision activity below) he gave them to resolve and thereafter present the solutions to the class
Group 1, 2 and 5 among the five, participated actively in this revision lesson.

Group 1 discussion

Learners in this group asked the teacher questions related to the revision lesson especially the trigonometric ratios. One of the learners in the group asked, "Why are we having three trigonometric ratios in Grade 11 and 12 but six in Grade 10?" The teacher did not answer this question directly but responded, "The reduced trigonometric ratios are the inverse of the remaining three, cot$\theta= \frac{1}{tan\theta}$ sec$\theta=\frac{1}{cos\theta}$, and cosec$\theta=\frac{1}{sin\theta}$." The teacher further explained that the reduction of the trigonometric ratios from six to three would help make it easy for the solving of trigonometric topic task in Grade 11 and 12 for learners.

Group 2 discussion

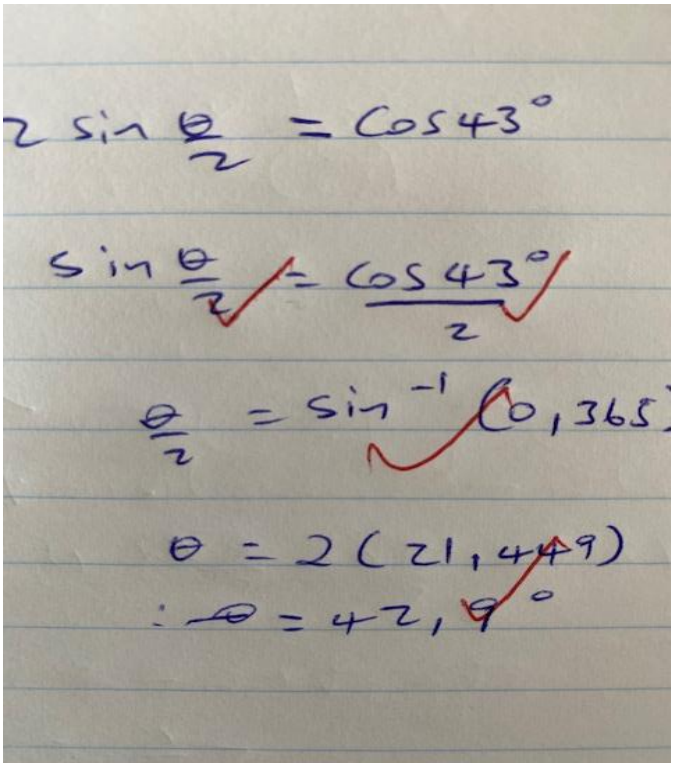
In this group, the learners questioned each other's contributions or responses to the revision activity. One learner asked: "Why do we have to use a calculator to substitute $sin45°$ to get $\frac{1}{\sqrt{2}}$ when the instruction said without using a calculator?" This question motivated a discussion about following instructions and the purpose of instructions as the teacher explained that it was important for learners to follow instructions to avoid being penalised. One other learner responded to the question and suggested to their group mate, "I think we should use special angles, especially when we have 30°, 45°, and 60° trigonometric ratios, meaning we should remember how they are represented as we were taught in Grade 10."

The teacher then drew a table showing the special angles, and advising learners to memorise it to respond to such tasks (revision activity).

θ	0°	30°	45°	60°	90°
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

Group 5 discussion

The conversation in this group focused on mathematics proficiency as learners used the cooperative learning opportunity to unpack and follow mathematics procedures (procedural fluency) when solving the revision questions. Learners in this group were able to solve 1.2 accurately; as they were able to divide both side by 2, and make θ the subject of the formula and accurately rounded off the answer as per instruction (see below).



The section that follows presents how the teacher implemented revision activities within their understanding of cooperative learning.

Revision activity

1.1 Simplify without using a calculator:

a) $2sin45° \ X \ 2cos45°$
b) $cos^2 60° \ - \ sin^2 60°$
c) $\sin 60\ ° \cos 30° - \cos 60°° \sin 30° - \ \tan 45°$
d) $4 \sin 60° \cos 30° - 2 \tan 45° + tan60° - 2 \sin 60°$

1.2 Given $2sin\frac{\theta}{2} = cos43°$, for θ E (0; 90°), determine the value of θ(correct to one decimal place).

"These cooperative learning activities that I am giving them challenge them to ask each other questions and to listen to what their peers are learning," said the teacher.

From the exercises, this study discovered that the learners were able to guide, encourage and learn from one another to understand ratios (see group 1, 2 and 5's responses). In addition, the teacher thought that group work was valuable. The cooperative learning value was that learners became comfortable in handling the topic with their classmates. Learners spoke more in groups compared to when the teacher interacted with them. This was consistent with findings from studies on the effect of cooperative learning and has also been investigated in other subjects (Wachanga & Mwangi, 2004). The feedback from the interviews showed that when learners were taught trigonometry through the cooperative learning approach, they were likely to be more receptive of knowledge (Wachanga & Mwangi, 2004).

After the cooperative learning exercise, the teacher asked each group to stand and give an answer to the problems given. The teacher also randomly picked the group mates whom he asked how the rest of the group approached the problems to produce the answers given (see observation discussion 4.4.1).

While the exercises with the learners pointed to positive outcomes from cooperative learning, the teacher bemoaned that group work consumed time, since more time was needed for discussions, collegiality and presentation of work to the whole class. The teacher also said that some learners would not participate if the teacher did not strictly supervise. This study found that supervision was deemed unproductive because the cooperative method emphasises the learners' independence during the learning process. The teacher said:

The classrooms are mostly overcrowded with at least 50 learners in one classroom, and that makes it hard for learners to sit in groups. The government should intervene for a maximum of 30 learners to be in one classroom to make learning conducive: allowing other teaching methods to be easily implemented. Also, teachers should undergo a workshop on other teaching strategies like cooperative learning for these methods to be effectively and well used in the classroom for the betterment of learners' performance.

The teacher seemed to believe that cooperative learning would be more successful in closely managed groups in less overcrowded classrooms. Overcrowded groups were viewed as inconvenient for focusing on struggling learners. However, these findings were not very divergent from the running theme in this study because indeed the literature revealed findings consistent with this critique of cooperative learning (Good, Reys, Grouws, & Mulryan, 1989-90; King, 1993; Mulryan, 1992, 1995).

The empirical evidence shows that there might be limitations to the usage of cooperative learning in trigonometry. Also, there may be interruptions in the learning process when group strategies are employed, and more time spent in rectifying these challenges. Participating in-group settings may succumb to complacency, especially the learners that are predisposed to the innate challenges of a course such as a trigonometry (Good, Reys, Grouws, & Mulryan, 1989-90; King, 1993; Mulryan, 1992, 1995).

Given the literature available, the limitation to cooperative learning investigated is time limits required for young learners to establish trust in groups and bond with peers. This was a common drawback to learners adapting to trigonometry as well as the accompanying learning materials (Good et al., 1989-90).

From the findings, these drawbacks are comparatively benign to other researchers who found significant benefits in learning trigonometry using cooperative learning and the psychological as well as social advantages. The capacity of learners to respond to trigonometry instruction in a group setting, as well as the individual setup, are instrumental to the effective understanding of trigonometry (Slavin, 1998).

We, teachers, use different methods of teaching, but as I said earlier, I believe in mixing these teaching methods to enhance understanding of learners in mathematics in general. I use all the methods of teaching in my classroom but cooperative learning is better than the traditional textbook method since it is learner-centred. It makes learners independent of their learning.

4.3.1 Summary to research question 1

The analysis of research question 1 is that the teacher agreed with the literature with regards to the view of cooperative learning. The teacher believed that cooperative learning provided equal opportunity to work on problems and exercises, and the opportunity helped the learners to learn from one another on how trigonometry problems were solved. Therefore, for the teacher, cooperative learning has the potential to produce better learning outcomes for learners. Through observations, the researcher checked whether the teacher implemented his view of cooperative learning in the classroom setting. The study found that the teacher viewed cooperative learning and applied it in the classroom to a larger extent. While there is a bias for mixed learning approaches to teaching trigonometry, which includes cooperative learning, the teacher was not very enthusiastic about cooperative learning. This might have stemmed from the fact that classes had more learners and effective interpersonal activities might not have been pragmatic for the teacher. Therefore, this necessitates the question of the cooperative learning opportunities available for students in trigonometry as discussed below in research question 2.

4.2 RESEARCH QUESTION 2

What are the cooperative learning opportunities for students in trigonometry?

This part presents and analyses the findings from the interview that was conducted with a mathematics teacher under the following subthemes: cooperative learning opportunities for students in trigonometry, and the perceived causes for the poor performance of learners in trigonometry. There are some references to learner participation because they are embedded in the responses by the teacher as shown below.

4.2.1 Cooperative learning opportunities for students in trigonometry

From the onset, the teacher said that they ensured that in every learning topic, learners were given a chance to form groups of six to work out answers to tasks. This helped him assess whether they understood a new topic or an old topic that he would be worried if they remembered how to answer questions.

I regularly put the learners in groups so that they can work out the questions on each new topic. Here, I require them to solve some basic questions at first and then exponentially give them more challenging questions. This allows them to settle and like the group work because the first set of questions will be easy.

The trigonometric topics are linked or integrated, for learners to use reduction formula to simplify trigonometric problems they should have a detailed comprehension of trigonometric ratios (see 4.2.1 Group discussion 1). The application of revision activities (Grade 10 content) is important for solving Grade 11 trigonometric problems. For example, using a special angle table or knowledge to simplify problems when using reduction formula to reduce equations and expressions when simplifying.

The teacher explained how he provided opportunities for learners to learn cooperatively. The teacher said:

What I tend to do is to make the learners use charts to illustrate the problem-solving exercises. I allow for learners to do presentations after solving the problems in the group settings to reveal their answers in front of the class. That way learners can write down for each other on the question being discussed and how the group is arriving at the answers that they would later present to me and the rest of the class.

The teacher's comment illustrates how cooperative learning is applied in the classroom so that learners can find it interesting to learn and understand. The teacher said that they liked to have learners read in groups in advance on a new topic so they would have had some understanding of what would be discussed in the class.

He said:

I like for the learners to go into groups and get ahead of what I am going to teach them so that they already grasp some of the important aspects of the new topic that we will *be learning. In addition to that, I ask them to come up with examples of what they have learnt to illustrate to each other how to solve clearly problems.*

The exercises carried out before classes and exemplification of concepts show how cooperative learning is made into a participative process. Learners were allowed an opportunity to grasp concepts before the lesson. They were also exposed to ample examples to engage the teacher with the new trigonometry concepts being introduced to them. In explaining the opportunities that he gave learners for cooperative learning, the teacher said he was reluctant to use it almost exclusively at times:

The weakness of this method, however, is that some learners do not go and read, and it becomes hard to tell in the class whether they are having difficulties or they just did not read in advance.

The teacher said that trigonometry could become easier for learners when they work in groups.

I make them rotate in the groups for each to have an opportunity to present something on the 3 trigonometric ratios (Sin, Cos and Tan). This rotational approach means that the learners also equally get an opportunity to present on all the ratios without only being allowed to focus on the one that they viewed as easier.

The teacher added:

I am very careful not to put fast learners in one group. I ensure that at any given time, the learners that can understand the topic better are grouped in with those having difficulties. I use my experience to judge those who understand better and those lagging.

The teacher also believed that cooperative learning worked better when the groups of learners were carefully managed. The teacher constantly analysed the performance of groups to check if every member of the group benefitted from the engagements that were carried out in the groups. In light of this feedback from the teacher, the researcher probed into some of the causes of the poor performance of learners in trigonometry.

4.2.2 Perceived causes for poor performance of learners in trigonometry

One commonality between the findings from the learners and the teacher was that they both believed that lack of practice culminated into poor outcomes among learners in trigonometry. The findings in this study showed that the teacher believed most learners appeared to understand during the process of teaching and learning but performed poorly during tests and examination. This study further revealed that poor performance in mathematics as a whole could be attributed to learners not having study groups for mathematics practice and other subjects.

This finding is backed by empirical evidence through experimental research conducted by Ugwuadu and Abdullahi (2012) in Nigeria, using the cooperative learning method. It revealed a significant difference between the mean achievement scores of experimental and control groups in favour of the experimental group. Cooperative learning approach resulted in significantly higher mean achievement scores in biology compared to regular teaching method (Muraya & Kimamo, 2011).

4.2.3 Summary to research question 2

The teacher provided cooperative learning opportunities through illustrations using charts. The teacher also leaned towards constructivist practice by allowing more participation of learners through presentations during problem-solving activities. When it comes to the discussions about the challenges in learning trigonometry, the learners and the teacher agreed that lack of practice led to poor outcomes among learners in trigonometry.

4.3 RESEARCH QUESTION 3

What is the effect of cooperative learning on students?

The discussion below for research question 3 is drawn from the thematic findings from the learners that were interviewed and observed. This discussion was carried out under the following sub-themes: the views of group work in a classroom; the difficulties in learning and understanding trigonometry; the methods that are perceived to be effective in learning trigonometry for understanding; and the effect of using cooperative learning in a mathematical classroom.

The discussion also extrapolates from the data among learners' interviews on the perceived difficulties in solving trigonometric problems/questions; the mathematical resources used for the learning of mathematics besides textbooks as well as the nature of the impact of cooperative learning in enhancing learners' conceptual understanding of trigonometry.

4.3.1 The views of group work in a classroom

An illustration of views on group work in a classroom is as follows:
One of the learners (**S7)** said:

Group work for me means learning mathematics within a group with the other learners. In the group, we help each other understand the trigonometry questions that are being asked by the teacher and we help each other provide answers to the questions. This group work is different from when we are given tasks and told to solve them without the help of others. The group work means we have to interact with others and a lot of us contribute to the work that we are given as opposed to working as one person.

The learner viewed group work as an opportunity to learn from others. This paper argued in the literature review that group work implemented through cooperative learning instilled necessary social skills that were in any case relevant to the wider learning environment. In addition to this, the findings showed that aspects such as adolescence could interfere with learners' proficiency in otherwise challenging courses such as trigonometry. **S6** said:

Group work means going into groups and working with each other to understand better what we have learnt. The ones that can understand the topic can explain to those who cannot understand. For example, when we are learning the ratios, my group members *can explain to me how to go about using Sin and I can explain to others about Cos because that is what I am good at.*

This study argues that learners who struggle with trigonometry can begin to cope and develop a better understanding of concepts through the interpersonal relationship opportunities that are presented in cooperative learning as supported by findings from researchers such as Wood (1987).

Another learner (**S3)** said:

My view of group work is that of learning mathematics together in the classroom without the teacher involved. It is a time when we learn with each other and from each other. We look at questions that we are given on our own and we find the answers as a group. I have challenges with understanding the Cartesian plane so the members of the group can help me with that during the group work activities.

Another learner (**S2)** agreed:

Group work is also where we explain to each other what is required to answer trigonometry questions and help others who do not understand how to answer questions. Group work is also a place that allows me to learn how to answer trigonometry questions that I have challenges with answering by myself. Group work makes us not compete but forces us to want to all succeed. During a group work session, the other learners in my group struggled to draw triangles to show the four quadrants from 0° to 360°. I helped them and taught them how to do it, I also taught how to use SOHCAHTOA.

Group work meant learning from one another for the learners as well as the involvement of the teacher in developing a better understanding of a concept through prescribed activities. It was noted in the literature that the cooperative learning approach has been studied in the context of its correlation to other beneficial psychosocial or psychological results. One of these results is the quality of interpersonal relationships that develop during group work (Slavin, 1984). The argument and, by extension, the contribution that this study makes is that cooperative learning induces the necessary team experience that may, in the long run, be crucial to tackle learners' challenges in learning the intricate aspects of trigonometry.

To back up these claims, researchers such as Augustine, Gruber and Hanson (1989) have also researched the interpersonal relationships that group work brings to the classroom and the findings from this study are consistent with theirs when it comes to the positive perception of learners towards the value of interactions in the group. This study affirms this characterisation as consistent with constructivism, which theoretically underpins this study. This is because constructivism emphasises the active role of learners in building and making sense of knowledge. Constructivism moves hand in hand with learning in groups because it lays the foundation for cooperative learning which substantially enhances the conceptual understanding of trigonometry (Jordaan, 2016).

S9 agreed with **S3,** saying group work made the learners work together, not compete but yearn for collective success. They argued that due to the cooperative nature of group work it became the goal of the whole group to ensure that everyone succeeded. This finding from the learners described positive interdependence in the theoretical groundings of cooperative learning. Positive interdependence is crucial to cooperative learning (Johnson & Johnson, 1987). The learners must believe that their success depends on the success of their counterparts and that the only way they can achieve higher scores is when everybody succeeds.

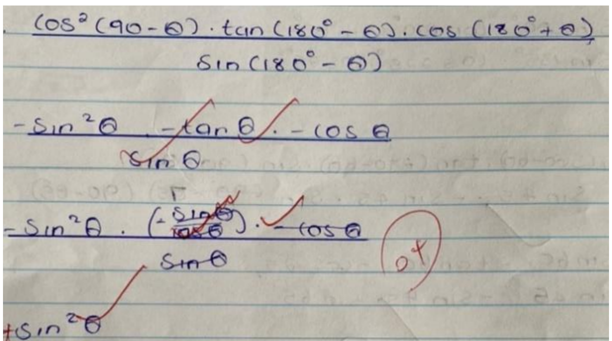
This may help to better understand the sentiments of the learners interviewed. This is because when undertaking trigonometry, the principle of interdependence implies that all learners must work together for a common goal of succeeding in the activities. The learners can then ensure that each person in the group can articulate the answer that is agreed upon and, thereafter, the group can tackle the assigned responsibilities (Johnson & Johnson, 1989). In light of this literature, the learners were asked about some of the benefits of using cooperative learning in a mathematical classroom.

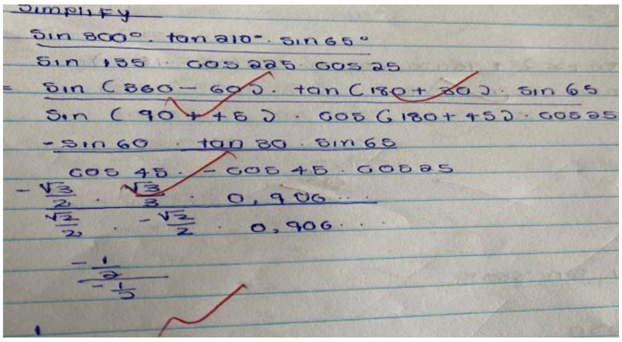
4.3.2 The effect of using cooperative learning in a mathematical classroom

Findings of this study can reveal that the opportunity to learn from one another is one of the learners' benefits of engaging in cooperative learning. **S3** asserted:

Cooperative learning allows me to learn from the other learners because those who are quick to catch what is being said in class are good at explaining to me how to solve problems. Some of the learners are also good at explaining the Cos and other ratios. I like that opportunity to learn the rations one-on-one unlike when the teacher is explaining them to the whole class.

The learner was able to grasp a topic that was complicated to them through cooperative learning. This finding is congruent with empirical studies from the literature which investigated the effects of cooperative learning on learner achievement and attitudes in a secondary mathematics classroom were investigated by scholars such as Whicker, Bol & Nunnary (1997). They used a quasi-experimental design to compare two pre-calculus courses; learners in class 1 studied the material in cooperative learning groups while learners in class 2 studied the material independently. Three-chapter tests were used to measure learner achievement, and a questionnaire was administered after the study to assess learners' attitudes towards the cooperative learning procedure. Learners in the cooperative learning group had increasingly higher test scores than learners in the comparison group and significantly outscored the comparison group on the third chapter test. Most learners indicated that they liked working in groups and appreciated getting help from other learners, especially for learning difficult concepts. Below are snapshots that show the responses from two groups, showing that learners understood the activity they considered difficult before working on the solutions in groups (a) from group 2 and (b) from group 4 respectively.

a) 

b) 

Another learner said that cooperative learning inspired good interrelations between classmates, meaning that they could easily approach one another with any learning difficulties.

Cooperative learning is good because we even get to talk with other classmates that we do not always talk to. In talking we get to realise who is good in trigonometry, then find what they are doing right to pass. I am good at solving algebraic equations since the knowledge is required to solve and simplify trigonometric questions. So, when we are in groups, I can show my colleagues how to use it when they are experiencing challenges.

If we can get along and live in the neighbourhood, it means we can also form a team and study after school and on weekends.

All the learners appeared to appreciate the interpersonal communication that came with cooperative learning as an approach to understanding trigonometry better. This study can also reveal that the studies on the impact of cooperative learning have also pursued other variables besides that of achievement. The cooperative learning approach has been studied in the context of its correlation to other beneficial psychosocial or psychological results. One of these results is the quality of interpersonal relationships that develop during group work (Slavin, 1984). The argument can be broadened in the context of this study to articulate that cooperative learning induces the necessary team experience that may, in the long run, be crucial to tackle learners' challenges in learning the intricate aspects of trigonometry, for instance (Augustine, Gruber & Hanson, 1989).

Two other learners said that cooperative learning was a confidence booster for them because it allowed them to express themselves in the small groups of three to four before going big and making contributions to the teacher in front of the whole class.

Cooperative learning is good because I get to say what I may be afraid to say in front of this whole class. It is better to get a negative response from the small group than the whole class because sometimes I feel shy to say that I have challenges with solving algebraic equations in front of everyone, especially when we have now moved to a new topic where I need to know equations to do well there.

Another learner added:

Being in groups is better because the teacher tells us to listen to all the group members. In one of the groups, I can see that the group members are struggling with SOHCAHTOA and then I feel confident to indicate that I also have hardships with it, and those who know can help with that difficulty.

Cooperative learning gave learners a space for expressing themselves and a platform to listen to one another. In the literature, there are results similar to findings of this study, such as those of Flynn (2013) who examined the effects of cooperative learning on the academic achievement, classroom behaviour and attitude of learners in mathematics. The effects of cooperative learning show the following: (a) a positive correlation between cooperative learning and increased test scores; (b) with the proper interventions, cooperative learning can help learners learn how to be members of a cooperative learning community; (c) a cooperative learning environment can be beneficial to learners who have previously had bad experiences with the subject; and (d) there are a variety of ways that cooperative learning can be used in the classroom. He further claims that cooperative leads to success in the general mathematics classroom.

Six learners in the current study agreed with this characterisation, adding that cooperative learning helped them correct one another and accept their classmates' inputs on solving mathematics problems. Cooperative learning also had the quality of independence, which learners liked a lot. According to them, it allowed for learning independently without the teacher "spoon-feeding" them.

4.3.3 The nature of the impact of cooperative learning in enhancing learners' conceptual understanding of trigonometry

Learners were interviewed within groups consisting of at least four members and one educator in Grade 11 mathematics classrooms.

The same interview questions were used to get their understanding, opinions and beliefs of learning in small groups than in the traditional textbook method. The Grade 11 teacher and learners were interviewed to get more understanding of how they feel and think of the use of cooperative learning in a mathematics classroom.

Some of the learners said that cooperative learning was not common in their classroom and the three weeks they were exposed to it, it was helpful for them to learn from other people in the classroom. Another running theme among the learners was that the cooperative learning exercise emboldened them to express themselves during problem-solving. The cooperative learning exercises made them realise what they were getting wrong most of the times because the interactive session exemplified concepts for them.

> *In a way, some of the biggest problems that I had before the exercises are gone because I can see this is not as difficult as I thought. Although I am still catching up, I now know where I get it all wrong. The cooperative learning exercises allowed me to fix my difficulties and I wish we had more of them every time we have a new trigonometry topic because I tend to get lost at the beginning, which means I have a difficulty following through as we finish a chapter of the topic we are being taught by the teacher.*

Cooperative learning helped some learners overcome the complications they faced. One other learners reiterated what had been noted before that the cooperative learning helped *“built my self-confidence, as I could comment on the work written on the board by other groups.”* The findings from the literature are congruent to this because they have linked cooperative learning to increases in self-esteem, attendance, time on task, enjoyment of school and classes, and motivation to learn as well as a decrease in dependence on the teacher (Augustine et al., 1989-90; Good, Reys, Grouws & Mulryan, 1989-90; Slavin, 1990).

Another learner (**S4)** described the nature of the impact of cooperative learning in enhancing learners’ conceptual understanding of trigonometry.

> *Trigonometry has been difficult for me in Grade 10, but this time using group work made me understand the trigonometric ratios and reduction formula to simplify trigonometric questions.*

Others reckoned that learning in groups needed enough time and space. The class should not be overcrowded and our teachers should have effective ways to facilitate cooperative learning.

4.3.4 Difficulties in learning and understanding trigonometry

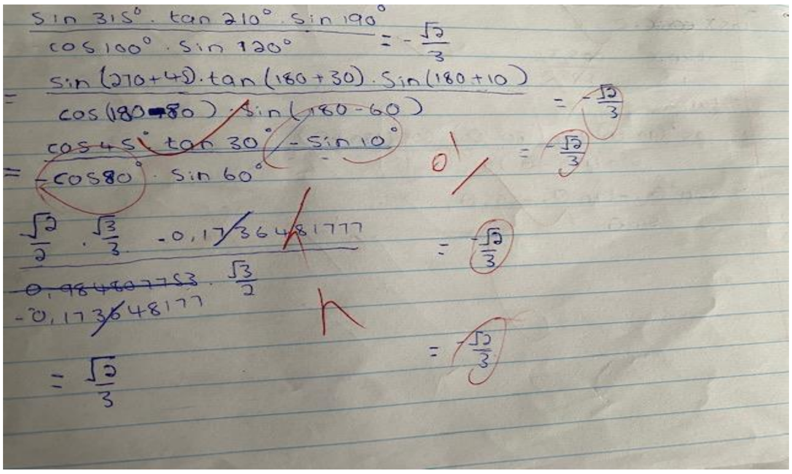
While the evidence suggests that this method enhanced learners’ knowledge of trigonometry, knowledge lasted only for a moment. This study treads on the argument that this state of affairs can prove to be problematic because learners do not retain any knowledge eventually.

One of the learners **S1** said:

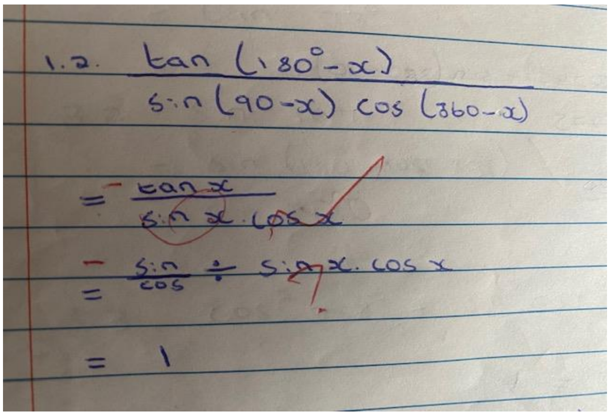
The challenge that I face with learning trigonometry is that the subject requires me to have a lot of understanding. This is especially in my case when the problems given by the teacher require me to simplify. It has taken me a while to grasp that but the more I get confused with simplifying, the more it creates problems for me during homework and tests.

The learners’ comments point to the complexity of trigonometry as a challenge. The findings from the study showed examples of research by Orhun (2002) that found difficulties faced by learners in using trigonometry when solving problems. The existing body of knowledge found that the learners did not develop the concepts of trigonometry clearly. The literature review showed that learners made some mistakes, and the existing body of knowledge attributes these challenges to the teacher-active method and memorising methods.

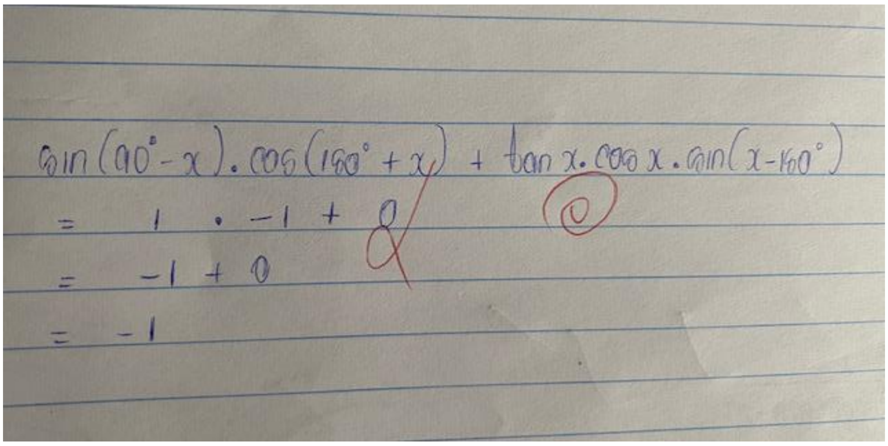
The following are some of the examples of the mistakes the researcher observed in the trigonometric classroom.



a)



b)

c) 

There were more difficulties that were raised among the learners and another learner (**S11)** said:

> *I have a problem with solving for x. This is a problem for me because questions such as that are asked in the class all the time and I end up feeling discouraged and sometimes frustrated because I do not know how to solve those problems. I also have a difficulty when I am asked to factorise because I cannot easily follow what is being asked of me. In these two issues with finding x and factorising, I just end up confused and failed in assignments and test. The knowledge of solving for x and to factorise helps with solving trigonometric related tasks (Annexure 1)*

This study found that this was a somewhat common challenge among the learners and they articulated these difficulties clearly. These were deep-seated concerns for the learners and they had contemplated these difficulties for a long time. Another learner (**S6)** gave feedback on the difficulties they faced:

> *I have a big problem using sinθ, cosθ and tanθ. I think they make trigonometry complicated for me very much because it is just a lot of things to learn and then keep in mind because you need to have that knowledge every time. This makes it difficult every time I am given assignments or tests. During exams, I have to study this and spend most time on it because it is so challenging and I need to go into the exam knowing how to solve the problems. I try to study how the three relate because I think the understanding of the relationship among that is very important.*

The learners experienced challenges with using sinθ, cosθ and tanθ. This finding is consistent with the findings from the literature, that there were challenges among learners in navigating problems that contained cos, sine and tan in trigonometry. For example, Brown (2006) studied learners' understanding of sine and cosine. She reached a fragment called trigonometric connection. Brown's study shows that learners had an incomplete and disjointed understanding of the three major to view sine and cosine as the coordinates of a point on the unit circle, as a horizontal and vertical distance that are graphical entailments of those coordinates, and as ratios of sides of a reference triangle. Some learners said that they had challenges applying the reduction formula, trigonometry was not taught for understanding in Grade 10 and that has an impact unto understanding the trigonometry of Grade 11 since the curriculum is progressive.

The findings are consistent with the argument that was found in the literature that the common errors and misconceptions exhibited by learners in solving trigonometry problems are among others the incorrect substitution. Learners struggle with reduction formulae, especially with the signs of the reduced trigonometric ratios, while some learners could not apply co-functions correctly (National Senior Certificate Diagnostic Report, 2017).

Therefore, learners lack conceptual understandings of trigonometry. In addition, Gur (2009) investigated the types of errors, underlying misconceptions and obstacles that occurred in trigonometry lessons and found congruent results to the findings in this section of this study. He used the sample of 140 Grade 10 high school learners and 6 Grade 10 mathematics teachers that were observed. The study used a diagnostic test consisting of seven trigonometric questions, the learners' responses were analysed and categorised.

Gur (2009) found that most common challenges that learners experienced were improper use of equation, order of operations, and value and place of sin, cosine, misused data ($tanx. cotx$ multiplication always gave $1 = 1$), misinterpreted language (misconceptions related to a concept that produced a mathematical object and symbol. For example, "I did not know which side of the triangle is called the adjacent edge or opposite edge"), logically invalid inference ($sinx\ and\ cosx$) are only defined in a unit circle; $sin^2x + cos^2x = (sinx - cosx)^2 + (sinx + cosx)^2$; $tanx\ and\ cotx$in relation to each other are complete at 360°and it is the same for every value), and technical mechanical errors (the equation is equal $1 - 2cos^2x$; $a^2 = 9 + 16 - 8a + a^2\ then\ a = \frac{25}{8}$; $tan2A = \frac{3}{4-a} = \frac{3}{4-\frac{25}{8}} = \frac{24}{7}$). The challenges among learners in this current study were associated with equations and finding the value and place of sinθ, cosθ (NSC Diagnostic Report, 2014).

The literature argued that conceptual understanding is the ability to solve both lower and higher order trigonometric problems correctly, being able to make connections of previous lesson to apply the acquired knowledge in various aspects of trigonometric tasks. When a learner can proof the following identity $sin^2x + cos^2x = 1$, then the learner will be able apply the knowledge to calculate $sin^2(43^{\circ}) + cos^2(43^{\circ})$.

One other learners brought a different view to their difficulties by invoking the inadequacies that surrounded the actual instruction of trigonometry in the school. **S2** said:

I think my difficulty is actually with teachers. I do not think the way they teach is enough because no matter how much information is explained to me; I am still left confused. I think that we lack more resources that can make learning trigonometry easier for us. We need more materials to help us understand trigonometry better so that if I do not understand what the teacher is saying, I can go to those materials and try to see if they can make me better understand. Those materials may be videos, maths props and textbooks that have good guides to explain step by step how to solve trigonometry problems,

The comments of the students above seem to relate trigonometry challenges with the teacher's approach of introducing concepts to them. Before delving into the actual content of the learners' difficulties here, learners did not view their teacher as approachable. For instance, the above learner makes a case of more resources and does not seem to recognise that the teacher can also be a useful resource for probing further about confusing problems in the classroom. The students suggested that the teacher could use videos, mathematics props and textbooks that have good guides to explain step by step how to solve trigonometry problems. Fortunately, this study was double-edged - it also had prepared to assess the teacher's approaches to teaching, which made for a good observation on whether teachers' methods of teaching may have been the root causes of learners struggling to solve equations, for instance.

The finding was consistent with the arguments highlighted in the literature that there was a breakdown between the teachers and learners, which consequently impacted badly on the performance of learner in trigonometry. This is described in Brown (2006) and Orhun (2002)'s findings which argued that the pedagogy or teaching method did not assure learners' in-depth understanding and memory retention of knowledge.

Another learner **S10** agreed with **S2** that how teachers taught was insufficient for them because the information could not be recalled. **S10** said that more group work needed to be encouraged in trigonometry classes to improve their trigonometry proficiency. Once again, learners did not see the teacher as the first point of reference whenever they needed assistance with the problems they experienced in the class. The suggestion about group work compounded the view of this study. However, cooperative learning can produce better learning outcomes for trigonometry learners and this finding validates that argument. Hence, the researcher in this study agrees with the findings of the scholars in the literature, concluding that an approach such as cooperative learning could enhance conceptual understanding, including memory retention, making connections and assure in-depth understanding of trigonometry.

Another learner (**S6)** said that their difficulties with trigonometry were due to:

> *Lack of practice and understanding. This is because trigonometry is there in Grade 10 but we were not well taught. This is one of the greater reasons why understanding trigonometry becomes a problem when it continues in Grade 11.*

S8 agreed, saying there was a breakdown in terms of the learning they received between the different grades, which presented challenges later on for them in Grade 10. For them, there was no coherence in the teaching, which created a void as well as continued confusion when they transitioned into the trigonometry class of the next grade. This finding exhibits the learning on constructivism that this study adopted as the theoretical underpinning. According to constructivism, learning is not an event and should be continuous. Findings of this study noted that learners reported that trigonometry teaching was not continuous and that subsequently presented difficulties for them. Robson (2006), Fraser (2006), and Troutman and Lichtenberg (2003) say the constructivist method of instruction recognises the importance of the learner in the learning process and they believe that learners should build their own understanding of concepts through self-discovery. Therefore, knowledge is not obtained passively but is actively and continuously adapted by structuring and re-structuring information and experiences as the learner develops to a higher level of understanding (Donald et al., 2006; Troutman & Lichtenberg, 2003).

The findings from this study found that the teachers were not aware of these practical implications. As mentioned in Chapter 3 and thereafter in 4.1, observations of the teacher during class was conducted to evaluate their teaching and interaction with learners.

4.3.5 Summary to research question 3

The findings from interviews carried out with learners show that the understandings of group work echo the theoretical interpretations of the learning approach. The findings revealed the opportunities availed for learners through the use of group work. Improved interpersonal relationship as a by-product of cooperative learning was revealed in the analysis of research question 3, confirming the literature findings from Slavin (1984) in Chapter 2, that cooperative learning significantly improves the quality of interpersonal relationships among learners. The majority of the learners said that cooperative learning exercises were beneficial. Therefore, this study noted from the analysis that cooperative learning was not common among learners in their classroom. However, when they were taught using the approach in some instances, their understanding of trigonometry improved.

The following section presents the observation this study carried out in the classroom.

4.4 FINDINGS FROM THE OBSERVATION GUIDES

The researcher used an observation guide in the mathematics classroom to evaluate the engagement between the teacher and the learners in the trigonometry class. Six classroom observations were conducted: two lessons were observed per week for three weeks in a Grade 11 mathematics classroom.

The following task form part of the trigonometric activities or tasks learners were given to solve: -

1. If $\sin 17° = a$, WITHOUT using a calculator, express the following in terms of a:

1.1. $\tan 17°$

1.2. $\sin 107°$

1.3. $cos^2 253° + sin^2 557°$

Week 1

The following revision activity (lesson one) is a continuation of the group discussions in research question one above.

Lesson one

Topic: Trigonometry revision (Grade 10)

Learners were placed in groups of 6, and there were 5 groups in the classroom. In the first lesson, the teacher after grouping the learners introduced the topic using revision activity of the six trigonometric ratios (sin ϑ, cos ϑ, tan, cot ϑ, sec ϑ, and cosec ϑ) learnt in Grade 10. The teacher further highlighted that those ratios are reduced to three (sin ϑ, cos ϑ and tan ϑ) in both Grade 11 and 12 respectively. There was an integration of activities from basic algebra to trigonometry e.g. solving $2x = 1$ (algebraic equation) to solving $2sinx = 1$ (trigonometric equation). The teacher emphasised the importance of in-depth knowledge of basic algebra in solving trigonometric tasks. Learners were given a home activity which had the Grade 10 revision and introductory questions of the derivatives and identity, to be assessed on the next lesson. The observations conducted in this study, after this lesson, were fuelled by the challenges highlighted by both the learners and the teacher with regards to learning trigonometry. The researcher observed the manner in which group work was employed, the teacher's approach to introducing trigonometry concepts and the efforts surrounding interactions between the learners and the teacher within the classroom.

Part of the observation shows that at times the teacher failed to adequately introduce concepts. Also, there was a lack of interpersonal interaction between the learners and the teacher, especially when it came to responses to queries about concepts. The observations are tabulated in the lessons below.

Table 4.1 Teaching and learning approach: Cooperative learning

TEACHING AND LEARNING APPROACH: COOPERATIVE LEARNING			
1	2 **X**	3	4
Learners did not work as a group.	Learners worked as a group, but roles were not allocated to group members.	Learners worked as a group for an activity and roles were allocated to group members.	Learners worked as a group throughout the presentation and roles were allocated to group members.
COMMENTS	Learners were seated in groups, but the cooperative learning was not effective as the teacher did not allocate any roles to group members.		

From this observation, learners worked as a group but the teacher did not allocate roles to group members. The finding is that the learners were seated in groups but the cooperative learning was not effective because the teacher did not allocate any roles to them. The interaction among group members is critically important to the success of small group activities (Shachar & Sharen, 1994).

The teacher may have not understood that interaction would only happen when teachers create conditions that enable learners to work in small groups on tasks that require cooperation among group members. In the classroom observed, opportunities for learners to work in situations where they experience positive interdependence will be a better choice than situations based on negative or no independence.

Lesson two

Topic: Derive and Use Identities

The home activity given on lesson one was assessed and learners shared their answers by writing on the board. The teacher's role was to comment and clarify each answer. The teacher introduced the topic of trigonometric identities to the class showing the learners how it was integrated to basic algebra.

The teacher also taught proving of identities $tan\theta = \frac{sin\theta}{cos\theta}$ and $sin^2\theta + cos^2\theta = 1$.

This was done with learners as an introduction. The researcher in this study observed that the learners were able to interact actively on the lesson and contributed during the process of teaching and learning of proving the identities.

The teacher gave learners an activity [to solve in groups and learners discussed the solutions when proving the identities]. The problem below is an example from the activity given to the learners: -

- Prove the following identity: $\frac{sin315°.tan210°.sin190°}{\cos 100°.sin120°} = -\frac{\sqrt{2}}{3}$

The learners had mastered the use of the reduction formula to simplify expressions (see 4.2.1 Group 5 discussion). At the end of lesson two, the teacher gave learners a home activity of proving basic trigonometric identities.

The table below presents some of the salient observations from lesson two.

Table 4.2: Essence of Trigonometry: Basic ratios and reduction formula

Essence of Trigonometry: Identities and reduction formula			
1	2	3	4 **X**
Trigonometry is not introduced.	Trigonometry is fairly introduced with revision activities from previous grade.	The use of Sin, Cos and Tan is used in the introduction but with few activities to enhance learners' understanding.	The lesson was well presented with Sin, Cos and Tan as ratios, and relevant examples to enhance leaners' understanding were given. The use of the mnemonic of SOHCAHTOA was used with enough examples.
COMMENTS	The lesson delivery on trigonometry was well done, the teacher demonstrated knowledge and confidence about the content of trigonometry.		

Regarding the essence of trigonometry - basic ratios and reduction formula - the lesson was well presented with Sin, Cos and Tan as ratios, and relevant examples to enhance learners' understanding were given.

The use of the mnemonic of SOH Sin $\theta = \frac{opposite}{hypotenuse}$ CAH Cos $\theta = \frac{adjacent}{hypotenuse}$ TOA Tan $\theta = \frac{opposite}{adjacent}$ was used with enough examples (see activities below, 1 a to e). The lesson delivery on trigonometry in this instance was well done. The teacher introduced the lesson using previous grade trigonometry and simple to complex examples when introducing topics.

Week 2

Lesson three

Topic: Use reduction formulae to simplify the trigonometric expressions

The lesson began with the assessment of the home activity given on the previous lesson and learners discussed and shared their answers but with inadequate understanding. The introduction of this topic was done using a Cartesian plane and the drawing of triangles to show the four quadrants (from 0° to 360°) according to the mnemonic SOHCAHTOA. The teacher solved the examples with the learners writing on the board for functions values of (180°±θ), (270°±θ) and (360°±θ). Learners were given an activity (see activity below) to do cooperatively in groups. Findings from the observation were that not all learners in the group were active. The observation also showed that the teacher failed to address the learners' reluctance to participate in cooperative learning although this problem was apparent during his walkabout in the classroom.

1. Simplify
   a. $\frac{\sin 300°.tan210°.sin65°}{sin135°.cos225°.cos25°}$

   b. $\frac{\tan(180°-x)}{\sin(90-x).\cos(360-x)}$

   c. $\frac{cos^2(90°-\theta).\tan(180°-\theta).\cos(180°+\theta)}{\sin(180°-\theta)}$

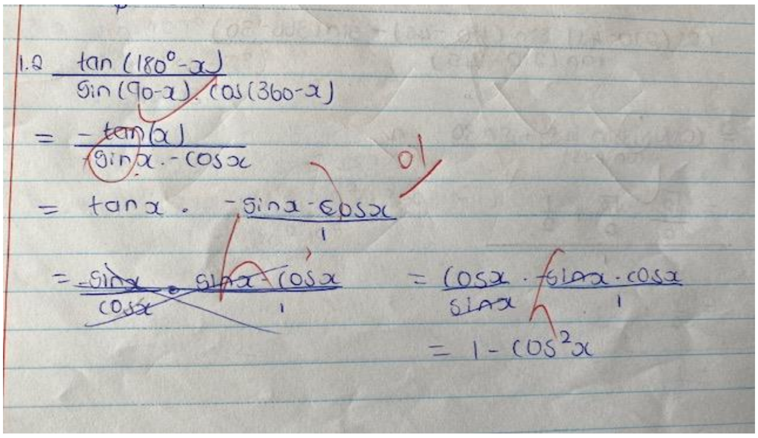
   d. $\sin(90° - x)\,.\cos(180° + x) + \tan x.cosx.\sin(x - 180°)$

   e. $\frac{\cos(-225°).sin135°+\sin 330°}{\tan 225°}$

In addition, learners were given a chance to share their answers on the board to the class. The observation was that this approach motivated members from other groups to comment and give their understanding on the answers shared by other groups. The teacher sat back while all the inter-group discussions happened, and he only came in to address general misconceptions such $\sin(90° \pm \theta) = sin\theta$. As with other sessions, the teacher gave learners a home activity as soon as the lesson ended.

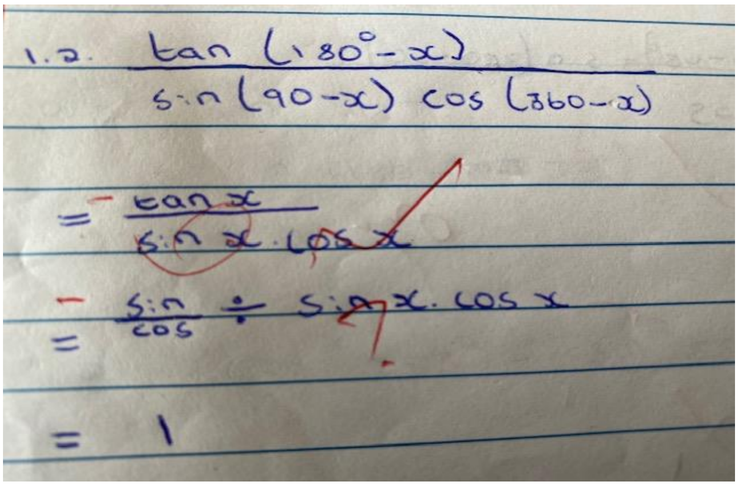
The challenge in this lesson was that learners had an inadequate knowledge of co-functions (see the responses below). Moreover, the tasks given to learners needed the knowledge of co-functions and the teacher omitted teaching them completely.

L1



The snapshot of this learner shows the response to 1.2 on the task given. The learner substituted $\sin(90° - x)$ with $sinx$, and it is supposed to be substituted with $cosx$. These type of mistakes contribute to the poor performance in trigonometry as the following step of solving the problem will not be correct.

L2



This L2 also had the same difficulty as L1 with substituting $\sin(90° - x)$ and the learner could not correctly substitute $\tan(180° - x)$, as he substituted it with $tanx$, and he was supposed to put a negative sign since $tanx$ is negative in the second quadrant.

The observation is listed in table 4.3 below with an analysis of the extent to which cooperative learning exercise impacted the learners' application and problem-solving skills.

Table 4.3: Strength of Trigonometry: Application and Problem-Solving

Strength of Trigonometry: Application and Problem-Solving			
1 X	2	3	4
The importance of trigonometry was not highlighted.	The importance of trigonometry was highlighted by showing its applicability.	The importance of trigonometry was highlighted by showing its applicability and mathematical notations were used.	The importance of trigonometry and its application was thoroughly highlighted and was integrated with science and engineering.
Comments	The topic was just introduced and the teacher continued with the lesson without its applicability.		

The observation can reveal that the importance of trigonometry was not highlighted. The topic was just introduced and the teacher continued with the lesson without explaining the importance of the topic in real life. Some learners contented that their difficulties were caused by the fact that teachers did not teach trigonometry in a manner that they understood. Such contention among learners could be alleviated by engaging learners about the importance of learning a concept, leveraging a learner-centred environment for better understanding.

Lesson four

Topic: Continuation of lesson three (derivation and the use of reduction formula)

The lesson started with the assessment of the home activity given to the learners on lesson three, the sharing of answers on the board was done as usual by the learners from different groups. There was a discussion about the procedural fluency of solving some questions as some learners got the same answers but solved the questions differently. During the observation, it was noted that the teacher was overseeing the interaction of the discussion and confirmed with corrections the solution to the given home activity to the whole class on the board. Further examples of simplifying trigonometric expressions and proving identities were done in the classroom by learners in groups and the teacher walked around the tables checking learners' answers and addressing misconceptions discovered.

The following table expressed this observation.

Table 4.4: Effectiveness in addressing or responding to trigonometric questions or problems

Effectiveness in addressing or responding to trigonometric questions or problems			
1	2	3	4 **X**
Teacher is ineffective at helping learners with problems or difficult task or questions.	Teacher generally tries to help learners who approach problems or questions but is not consistently effective at addressing these problems.	Teacher helps learners who approach trigonometric problems effectively but with inadequate explanation.	Teacher is consistently effective in addressing learners' questions, concerns and problems of all trigonometric topics: basic ratios and the use of reduction formula.
Comments	The teacher fully addressed learners' questions.		

Effectiveness in these observations was based on the teacher's ability to learners' questions, concerns and problems of all trigonometric topics. The teacher was consistently effective in addressing learners' questions, concerns and problems of all trigonometric topics: basic ratios and the use of reduction formula but there is no evidence to prove this. The teacher fully addressed the learners' questions. The observation contradicted the findings from interviews with learners because it appeared that learners did not view their teacher as approachable with these kinds of difficulties. For instance, during discussions of their difficulties with trigonometry, some of the learners said more resources were needed but did not recognise the teacher as a useful resource for probing further about confusing problems in the classroom. The common challenge to at least five learners in the classroom was the choice of a sign when reducing the trigonometric expressions like $\cos(\vartheta - 180°)$. Learners were given home activity with higher order questions, which needed conceptual understanding and procedural fluency of trigonometric problem solving.

Week 3

Lesson 5

Topic: Determine for which values of a variable an identity holds

The activities given in lesson five were assessed and five learners could not solve all the questions because of insufficient knowledge of special angles (15°, 30°, 45°, 60° and 90°). During the introduction of the topic, the teacher explained that the knowledge proving the identities was the pre-requirements for the conceptual understanding of determining the values of a variable the identity holds.

Also, that the teacher taught and showed learners the integration of proving the identities with determining the values of a variable an identity holds. Examples were done in the classroom on the board where the teacher explained the need for the denominator not to be equal to zero. Findings from the observation showed that the learners showed a high level of understanding by being actively involved in the lesson and the solving of given examples. Learners were given a worksheet with mixed activities for a classroom task and home activity. The observations focused on the extent to which the cooperative exercise promoted the active engagement from learners.

The table below explains the outcomes of the observation.

Table 4.5: Promoting learner's active engagement

Promoting learner's active engagement			
1	2	3 **X**	4
All the learners are not engaged in the lesson.	Most learners appeared distracted or disengaged.	Many learners were passively engaged, listening to or watching the teacher.	Most learners responded to teacher prompts, and/or actively manipulated materials.
Comments	There is a room for improvement in terms of engagement of learners in a lesson by the teacher.		

Many learners were passively engaged, listening to or watching the teacher. This observation is also consistent with the one made in Table 4.7 below which shows that learners rarely engaged in positive interactions with one another. Interaction among learners was moderate. At times learners sat together but did not solve the given tasks together. This is problematic because it implies that cooperative learning is not being implemented properly. Learners' participation is critical to cooperative learning as noted by Demitra (2006).

The steps of cooperative learning were said to be unclear for learners. Teachers argued that the process had not been optimal, leading to less successful mathematical problem-solving process in trigonometry. The overall view of this is that there is a room for improvement in terms of engagement of learners in a lesson by the teacher. The above problems can be minimised by a well-equipped learning and teaching strategy through proper on-the-job-training (Bot & Eze, 2016). There is a need for an investigation into the effectiveness of some of the innovative instructional methods, techniques and strategies particularly the application of cooperative learning in mathematics in secondary schools in South Africa.

Lesson six

Topic: Summary of derivation and the use of reduction formula

2. If $\sin 17° = a$, WITHOUT using a calculator, express the following in terms of a:

2.1. $\tan 17°$

2.2. $\sin 107°$

2.3. $cos^2 253° + sin^2 557°$

The lesson started with the assessment of the mixed activities which were given on the previous lesson and learners were having an active discussion as others were writing solutions on the board. The teacher also helped learners by addressing the misconceptions and emphasising on the important things learners need to be aware of when solving trigonometric problems: -

- Inability to solve complex trigonometric problems

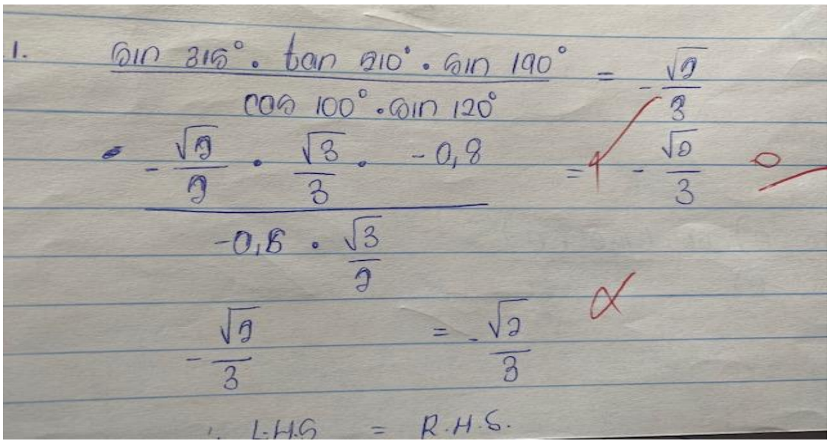
  Learners were able to replace $tanx$ with $\frac{sinx}{cosx}$ on the board and could not proceed further in proving the identity.
- Some learners could not factorise correctly.
- The division of zero when determining the values of a variable an identity an identity holds yields an undefined result.

The activity below is one of the proofs of the identity given to learners, it requires the reduction formula knowledge application, using special angles and being able to accurately solve expressions using algebraic knowledge.

- Prove the following identity:

$$\frac{sin315°.tan210°.sin190°}{\cos 100°.sin120°} = -\frac{\sqrt{2}}{3}$$

The following snapshots (responses to the activity above) were taken during observation and moving around the table of the groups.



1. $\frac{\sin 315° \cdot \tan 210° \cdot \sin 190°}{\cos 100° \cdot \sin 120°} = -\frac{\sqrt{2}}{3}$

$\frac{-\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{3} \cdot -0,8}{-0,8 \cdot \frac{\sqrt{3}}{2}} = -\frac{\sqrt{2}}{3}$

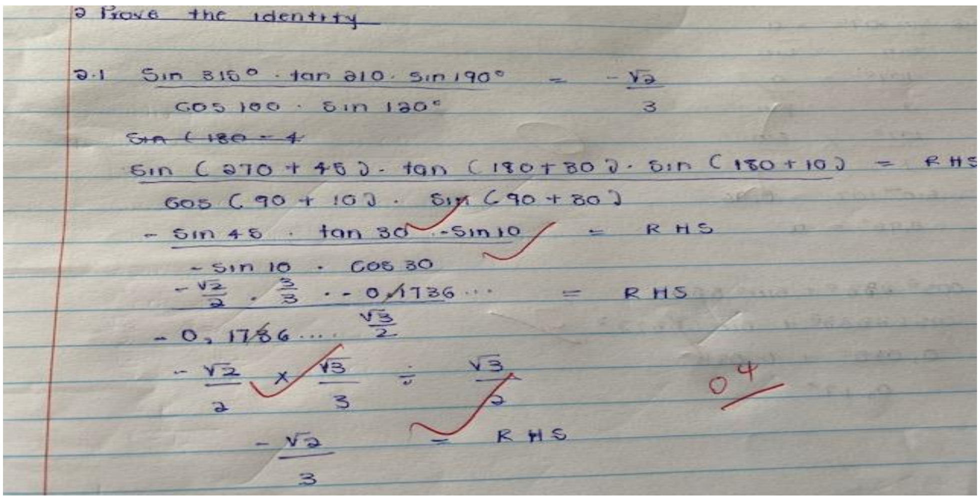
$-\frac{\sqrt{2}}{3} = -\frac{\sqrt{2}}{3}$

∴ L.H.S = R.H.S.

0

This learners' response is very poor as she could not use reduction formula to reduce the trigonometric terms, the learner just used a calculator to find answers ($\sin(315°)$ on a calculator equal to - $\frac{\sqrt{2}}{2}$.) . The learner was supposed to reduce $\sin(315°)$ to $\sin(360° - 45°)$ or $\sin(270° + 45°$, then get $-sin45°$ (and use special angle to get -$\frac{\sqrt{2}}{2}$).

L2



2 Prove the identity

2.1 $\frac{\sin 315° \cdot \tan 210 \cdot \sin 190°}{\cos 100 \cdot \sin 120°} = -\frac{\sqrt{2}}{3}$

~~Sin (180 − 4~~

$\frac{\sin(270 + 45) \cdot \tan(180 + 30) \cdot \sin(180 + 10)}{\cos(90 + 10) \cdot \sin(90 + 30)}$ = RHS

$\frac{-\sin 45 \cdot \tan 30 \cdot -\sin 10}{-\sin 10 \cdot \cos 30}$ = RHS

$\frac{-\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{3} \cdot -0,1736\ldots}{-0,1736\ldots \cdot \frac{\sqrt{3}}{2}}$ = RHS

$-\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{3} \div \frac{\sqrt{3}}{2}$

$-\frac{\sqrt{2}}{3}$ = RHS

04

This learner's response was mathematically proficient as the learner was able to use reduction formula to reduce trigonometric terms, and use special angles to substitute those ratios, the learner's solving problem was procedurally fluency. This learner is from Group 2, the group that was actively involved in the lesson during the revision activity as they consistently shown conceptual understanding when solving and discussing given tasks.

From this activity, the observation was how the teacher used cooperative learning to promote the interaction among the learners. The learners had the opportunity to contribute by asking questions and having the teacher redirect questions to the rest of the class for an answer. To his credit, the teacher limited his intervention only when necessary and invited learners for opportunities to engage. Observations from lesson six are tabulated in Table 4.7 below.

Table 4.6: Promoting interaction among learners

Promoting interaction among learners			
1	2 **X**	3	4
A teacher did not promote interaction among the learners.	Learners rarely engaged in positive interactions with one another.	There was an underlying positive tone to learners' interactions.	Learners were clearly positively connected to one another in planned activities to support their trigonometric understanding.
Comments	Interaction among learners was moderate: Sometimes learners sat together but did not solve given tasks together.		

4.4.1 Summary of the observation guide

The observations helped inform the study about the context in which the learners and the teacher viewed cooperative learning and how they perceived the challenges with learning trigonometry. The study used the exercises to learn first-hand experiences on the teaching and learning using cooperative learning and the essence of trigonometry with regards to teaching and learning basic ratios and reduction formula. The exercises helped to assess the application of cooperative learning towards problem solving in trigonometry. The researcher used the third lesson to observe and evaluate the effectiveness in addressing or responding to trigonometric questions or problems whereas the fourth lesson helped to observe how the teacher used cooperative learning to promote learners' active engagement as well as how the teacher promoted the interaction among learners.

4.5 RESEARCH QUESTION 4

What cooperative learning strategies can help improve students' competence in trigonometry?

The observations tabulated in tables 4.1 to 4.7 served as a point of departure to understanding how the teacher could enhance cooperative learning competence in teaching trigonometry and answer research question 4. Under research question 4, the methods that are perceived to be effective in learning trigonometry are discussed from the point of view of the learners as this study argued for a learner-centred approach learning of trigonometry in Chapter 2.

4.5 Methods that are perceived to be effective in learning trigonometry

The following sections describe preferences of learners regards learning trigonometry.

4.5.1 Cooperative learning approach to learning trigonometry

All of the learners interviewed said that group work offered through cooperative learning activities was the most effective way of learning trigonometry. They all preferred cooperative learning as a strategy for learning trigonometry. Learners seemed to derive most of their understanding of trigonometry through the opportunities for explicit engagement with their peers. One learner **S8** said:

> *I find that I am abler to learn about the thing that I do not understand when my classmates explain it to me. I prefer being in groups of three to four because*

then we are not destructed, and we all have a chance to contribute. I like this especially because it allows me to ask questions that I can ask in an open classroom.

The learners suggested that cooperative learning would provide an opportunity for them to learn trigonometry better. **S11** agreed that the group work allowed them to freely express themselves when it came to the possible answers that they worked for problems given in class:

I can say anything to my group because it is smaller compared to the whole class. I also feel like my difficulties can be heard because we will be in a small group, with a better chance of talking and asking others what are they thinking.

The learner suggested that learning in smaller groups was better than learning as a whole classroom. Regarding the face-to-face type of group work interaction, the learners argued the willingness to engage with others not only to benefit but also help other learners through cooperation. They say it encourages them to reorganise and restructure the information in their way and, in turn, develop clearer and more elaborate cognitive understandings than they held previously (Webb & Mastergeorge, 2003).

This finding on group work was important for this study because the study argues that cooperative learning can produce better learning outcomes in trigonometry. Cooperative learning is grounded in the belief that learning is most effective when learners are actively involved in sharing ideas and working cooperatively to complete academic tasks (Effandi & Zanaton, 2007). Furthermore, cooperative learning in a mathematics classroom involves social accountability, positive interdependence, individual accountability and groups' accountability (Kotsopoulos, 2010; Walmslay & Muniz, 2003).

4.5.2 Teacher-centred approach to learning trigonometry

Three other learners said that they learnt better when a teacher provided several examples and activities in the classroom. The learners argued that the exemplification of problem-solving helped them deeply understand the concepts at play and allowed them to replicate and even embed them. The learners followed problem-solving better when the teacher emphasised the most important aspects of given topics. Thereafter, the learners said they would be more positioned to follow the gist of the topic and experiment on their own to test if they had mastered the aspects taught. **S1** said:

> *I like it when the teacher writes all the things we need on the board, step by step. Thereafter, he emphasises how we should get the answer, and this helps me understand because I can see what is being done and hear from the teacher's words on how to solve the problem.*

Another learner added:

> *Sometimes the teacher leaves the content on the board which is good because I can look at it again and note that we have learnt something new. That way, I can remember it more. We can even also go to the board after class and explain to each other what the teacher was saying. So, writing on the board and giving more examples make me better learn and master trigonometry.*

One learner said that she learnt trigonometry better through using various resources to help understand trigonometry better, like study guides, previous questions papers and watching videos on the Internet.

4.5.3 Summary to research question 4

The learners interviewed preferred a cooperative learning approach as well as an interpersonal approach to teaching trigonometry, especially within small groups. Findings showed that group work and its benefits were viewed favourably. The findings also showed that deeper explanations under the auspices of repetitive explanations of concepts were favoured in both group work and through interactions with the teacher. The use of step by step explanations (described by **S1)** of concepts appeared to be one of the preferences from the learners.

4.6 CONCLUSION

This chapter discussed the themes that recurred from the data collected. The themes that emerged were the understanding of group work in a classroom; the difficulties in learning and understanding trigonometry; the methods that are perceived to be effective in learning trigonometry for understanding; and the benefits of using cooperative learning in a mathematical classroom. The other themes extrapolated from the data among learners were the perceived difficulties in solving trigonometric problems/questions; the mathematical resources used for the learning of mathematics besides textbooks as well as the nature of the impact of cooperative learning in enhancing learners' conceptual understanding of trigonometry. The next chapter presents the conclusion and recommendations of this study.

CHAPTER FIVE

SUMMARY OF THE STUDY, RECOMMENDATIONS AND CONCLUSION

5.1 INTRODUCTION

This chapter highlights the summary of findings of this study, which aimed to investigate and describe the use of cooperative learning in a Grade 11 classroom to enhance conceptual understanding of trigonometry. This deliberation is presented in section 5.2. Subsequently, section 5.3 presents the implications, followed by recommendations in section 5.4. The limitation of the study is discussed in section 5.5 preceding a concluding remark in section 5.6. The interview and the observations that were conducted with the teacher and the learners were resourceful in providing data used in the description of their experiences of cooperative learning in trigonometry.

The investigation of the study was guided by four research questions as follows.

- What are teachers' views of cooperative learning in the classroom?
- What are the cooperative learning opportunities for students in trigonometry?
- What is the effect of cooperative learning on students?
- What cooperative learning strategies can help improve students' competence in trigonometry?

5.2 SUMMARY OF FINDINGS

The summary of findings is discussed according to each of the research questions.

5.2.1 Research Question 1

What are teachers' views of cooperative learning in the classroom?

5.2.1.1 The main findings under research question 1

The key findings from research question 1 were that the teacher viewed cooperative learning as:

- an approach that gives learners equal learning opportunities,
- a way for learners to learn independently,
- opportunity for learners to ask questions, and
- as a method with which learners can actively participate in the classroom.

- **Giving learners equal learning opportunities**

The teacher viewed cooperative learning as an approach that afforded learners equal opportunities to work on problems and exercises, then learn from one another on how trigonometry problems are solved. He says cooperative learning gives learners equal opportunities and fair chance to work on problems and exercises and learn from one another to solve trigonometric problems.

The teacher also viewed cooperative learning as an opportunity for learners to learn from one another and solve trigonometry problems in a group setting. These findings were consistent with the interpretation prescribed in the literature (Jaelani & Retnawati, 2016, Wachanga & Mwangi, 2004); that is, cooperative learning is a learning approach that allows learners to work out mathematical problems together.

- **Cooperative learning as a way for learners to learn independently**

The teacher practised his view of cooperative learning by giving learners time to work out the trigonometry problems and then each was required to explain. During the lesson, learners were given a specified time to solve exercises and five minutes to share their answers on the board.

- **Opportunity for learners to ask questions**

The teacher viewed cooperative learning activities as challenging learners to ask questions among themselves and listen to what their peers are learning. He further described cooperative learning as a way to make learners participate effectively in the classroom.

- **Active participation of learners in the classroom**

The teacher interpreted cooperative learning as a way to make learners participate in the classroom. He viewed cooperative learning as resourceful towards reflecting on trigonometry problems. Learners corrected one another on the board when sharing solutions to the class. That helped them reflect on the mistakes and competencies they have when solving trigonometric exercises.

5.2.2 Research question 2

What are the cooperative learning opportunities for students in trigonometry?

5.2.2.1 The main finding for research question 2

The key findings under this research question showed that the teacher provided learning opportunities for learners through:

- illustrations using charts,
- presentation of answers to given activities on the board,
- placing learners in groups, and
- Solving questions from simple to complex.

- **Illustration using charts**

The teacher provided cooperative learning opportunities through the use of illustrations using charts. The teacher made the learners use charts to illustrate the problem solving that they were required to engage. The teacher argued that from his experience, this allowed the learners to write down for one another on the question being discussed and to highlight how the group arrived at the answers.

- **Presentation of answers to given activities on the board**

"I make them rotate in the groups so that they can each have an opportunity to present something on the 3 trigonometric ratios (Sinθ, Cosθ and Tanθ). This rotational approach means that the learners also equally get an opportunity to present on all the ratios without only being allowed to focus on the one that they viewed as easier," said the teacher.

The teacher leaned towards constructivist practice by allowing more participation of learners through presentations during problem-solving activities. The key findings from the observations in the classroom showed that after the ten minutes of cooperative learning, the learners were required to present to the class how they resolved the tasks given. The teacher subsequently required each the group members to contribute to the final presentation of cooperative learning outputs and demonstrate to the teacher that they understood how the group reached the answers that were revealed to the rest of the class.

- **Placing learners in groups**

The teacher gave learners a set of problems to respond among themselves. He placed learners in groups of six which were perceived to be enough to have a variety of contributions but also not too big for to the extent that other learners' contributions would be overshadowed by others. The desks were designed to face each other during cooperative learning, which created a face to face cooperation when learners worked in groups.

The teacher gave learners cooperative learning opportunities by regularly grouping them to work out the questions on each new topic. Learners sat in groups throughout the period of two weeks without being told to do so during the mathematics period.

- **Solving question from simple to complex**

The teacher required the learners to solve some basic questions first, then exponentially gave them more challenging questions. This from the teacher's perspective allowed the learners to settle and appreciate the group work because the first set of questions would be easy.

- **Practice of cooperative learning in classroom setting**

The observations helped to inform about the context in which the learners and the teacher viewed cooperative learning and how they perceived the challenges with learning trigonometry. The exercises were used to learn first-hand experiences of teaching and learning using cooperative learning and the essence of trigonometry with regards to teaching and learning basic ratios and reduction formula. The exercises helped to assess the application of cooperative learning toward problem-solving in trigonometry.

Outcomes of the observation:

- Learners worked as a group.
- The teacher did not allocate roles to group members.
- Learners were seated in groups but the cooperative learning was not effective.
- The teacher did not understand that interaction would only happen when teachers created conditions that enabled learners to work in small groups on tasks that required cooperation among group members.
- The importance of trigonometry was not adequately highlighted.
- Learners believed that the teacher did not teach trigonometry understandably.
- The teacher addressed learners' questions, concerns and problems of trigonometric topics: basic ratios and the use of reduction formula.

The problem with these outcomes is that teacher did not allocate any roles to the grouped learners. The interaction among group members is critically important to the success of small group activities (Shachar & Sharen, 1994). However, the teacher addressed learners' questions sufficiently at all times. The common challenge to most learners in the classroom was the choice of a sign when reducing the trigonometric expressions like $\cos(\theta - 180°)$. Learners were given home activity with higher order questions which needed conceptual understanding and procedural fluency of trigonometric problem solving.

5.2.3 Research question 3

What is the effect of cooperative learning on students?

5.2.3.1 The main findings for research question 3

The semi-structured interviews further provided more insight into the benefits of using cooperative learning in a trigonometry classroom elaborated under the following headings:

- Learners working together within their groups,
- Help each other understand difficult trigonometric questions,
- Learners' positive interdependence,
- Interpersonal interactions, and
- Improved interpersonal relationship.

- **Learners working together within their groups**

The learners perceived group work as a means of learning mathematics within a group with the other learners. Learners believed that cooperative learning availed them the opportunity to learn from other learners who better understood the $\cos\theta$, $\sin\theta$ and $\tan\theta$ ratios. They appreciated the opportunity to learn the ratios one-on-one unlike when the teacher is explaining them to the whole class.

- **Help each other understand difficult trigonometric questions**

The learners believed that the group provided an opportunity to help one another understand the trigonometry questions that were being asked by the teacher and they worked together to provide answers to the questions. The learners said that cooperative learning allowed them to learn from among themselves because those who quickly grasped the learning material were good at explaining how to solve problems.

- **Learners' positive interdependence**

The learners viewed group work as unique from individual tasks. They understood that their contribution and participation in given tasks were collectively and individually beneficial. Learners in each group had mutual trust. Solving tasks individually and sharing them with the group motivated them to work harder. The learners perceived group work as an opportunity to interact with others and to contribute to the work that they were given as opposed to working individually. Cooperative learning was perceived positively as an enabler to discuss with classmates whom they hardly had opportunities to interact with.

- **Interpersonal interactions**

Interpersonal interactions in cooperative learning enabled learners to identify peers who were good in trigonometry and, thereafter, found what they did right, followed it and passed. For instance, some learners said they fared better with solving algebraic equations and, in groups, they managed to show their colleagues how to use it when they experienced challenges.

- **Improved interpersonal relationship**

Improved interpersonal relationship as a by-product of cooperative learning was revealed in the analysis of research question 3, confirming the literature findings from Slavin (1984) in Chapter 2, which argued that cooperative learning significantly improved the quality of interpersonal relationships among learners. The majority of the learners said that cooperative learning exercises were beneficial. Therefore, this study noted from the analysis that cooperative learning was not common among the learners in the classroom. However, in few instances, they were taught using the approach and their understanding of trigonometry improved.

5.2.4 The nature of the impact of cooperative learning in enhancing learners' conceptual understanding of trigonometry

The perceived impact of cooperative learning revealed the following:

- Problems that they experienced before cooperative learning exercises were resolved because they could see within the group setting that algebraic equations and basic ratios were not as complex as they previously perceived them to be,
- Learners believed that cooperative learning exercises helped them figure out their mistakes when tackling individual tasks prescribed to them by the teacher, and
- Cooperative learning exercises provided the learners with an opportunity to confront the difficulties that they experienced with individual trigonometry tasks. One of the learners said that they wished they did more cooperative learning exercises every time the teacher introduced a new trigonometry topic. This was because learners often got lost at the beginning of new trigonometry topics, making it difficult for them to follow through the entire chapter of the topic they were taught by the teacher.

5.2.5 Difficulties in learning and understanding trigonometry

Two prominent difficulties in learning and understanding trigonometry were identified:

- The learners faced challenges with learning trigonometry because the subject required more time for them to grasp concepts and
- Learners had challenges with finding the value of x because it was a frequent requirement in trigonometry.

Learners also had problems with using sinθ, cosθ and tanθ. They attributed this to the fact that they were prescribed with learning materials which they had to recall and apply constantly. According to them, this hardened their attempt at assignments or tests.

Research question 4

What cooperative learning strategies can help improve students' competence in trigonometry?

5.2.6.1 The main findings for research question 4

The following findings were some of the strategies suggested for enhancing cooperative learning in teaching trigonometry:

- Cooperative learning approach to learning trigonometry, and
- Teacher-centred approach to learning trigonometry.

- **Cooperative learning approach to learning trigonometry**

Learners said that they learnt better when their classmates explained trigonometry solutions to them in a group setting. One student said: "*I prefer being in groups of three to four because we are not destructed, and we all have a chance to contribute. I like this especially because it allows me to ask questions that I may not ask in an open classroom."*

The learners said they felt that the group setting was conducive to learning trigonometry because they were smaller in number compared to an open classroom with the teacher guiding the learning process. Cooperative learning allowed learners to equitably participate in the classroom and feel freer to express themselves. This finding on group work was important for this study because the study argues that cooperative learning can produce better learning outcomes in trigonometry. This characterisation is consistent with constructivism, which theoretically underpins this study. This is because constructivism emphasises the active role of learners in building and making sense of knowledge. Constructivism moves hand in hand with learning in groups because it lays the foundation for cooperative learning which substantially enhances the conceptual understanding of trigonometry (Jordaan, 2016).

- **Teacher-centred approach to learning trigonometry**

Apart from cooperative learning, learners said that they also preferred a teacher-centred approach where the learning material was comprehensively explained to learners step by step. More traditional approaches of repetition were popular among the learners. Learners said that emphasis guided them through to understanding how the teacher arrived at solutions to trigonometry problems. Therefore, knowledge was not obtained passively but was actively and continuously adapted by structuring and re-structuring information and experiences as the learner develops to a higher level of understanding (Donald et al., 2006; Troutman & Lichtenberg, 2003).

5.3. IMPLICATIONS

5.3.1 Implications for practising teachers in the classroom

The implications of this study is that teachers need implement the South African national curriculum policy in order to offer cooperative learning opportunities effectively to learners (“the Laying Solid Foundations for Learning”). Strategies such as the use of charts cannot be a stand alone and should be accompanied by a combination of concrete illustrations, actions and words to make connections and facilitate learners’ understanding of abstract concepts in trigonometry. In addition, th findings from this study imply that cooperative learning should prioritise interpersonal relationships among learners in order to improve the solving of algebraic equations through group work. Thus, teachers should strive to implement teaching approaches that are recommended for Grade 11 classes in the South African curriculum and assessment policy statement. Teachers must be prepared to provide opportunities for learners to work in situations where they experience positive interdependence. This seems to be a better choice than situations based on negative or no independence (Shachar & Sharen, 1994). Teachers will be required to implement cooperative learning effectively by engaging learners about importance of learning a concept, then a learner-centred environment can be leveraged for better understanding. For the school environment, there is a need for an equal opportunity for cooperative learning as well as exposure to teacher-centred learning approaches.

5.3.2 Implications on the Department of Basic Education

The Department of Basic Education needs to organise in-service training to equip teachers with skills for innovative ways of providing cooperative learning opportunities to learners. Since teachers are faced with challenges in the implementation of cooperative learning in crowded classrooms, there is a dire need for presenting capacity-building workshops to address the challenges and enhancing the teaching of trigonometry. There is a need for a well-equipped learning and teaching strategy through proper on-the-job-training (Bot & Eze, 2016) as well as an investigation into the effectiveness of some of the innovative instructional methods, techniques and strategies – particularly, the application of cooperative learning in mathematics in South African secondary schools. Mixed approach should be used and the Department of Basic Education must provide guidance for teachers to evaluate the effectiveness of approaches used for teaching and learning trigonometry. The Department of Basic Education should also increase the training of teachers using cooperative learning. Workshops and training can involve small-group teaching approaches as well as methods to effectively engage learners during cooperative learning processes.

5.4 RECOMMENDATIONS

The findings of this study established gaps prevalent in the teachers' understanding of the practical implementation of cooperative learning practices. On the basis of the findings, the study recommends:

5.4.1 Teacher training

- Pre-service teacher training to be established as a major contributing factor to better the implementation of cooperative learning practices in trigonometry. Teacher education training institutions should emphasise the practical implementation of cooperative learning practices.

- Teacher training institutions to host practical workshops that would help teachers integrate their theoretical training with practical cooperative learning experience.
- Minimising the volume of trigonometry material and changing approaches weekly in order to engage groups of learners alternatively throughout the week in cooperative learning activities

5.4.2 Curriculum advisors

The findings of the study will assist curriculum advisors to create opportunities for teachers to participate in cooperative learning capacity-building workshops as follows:

- The curriculum advisors should encourage teachers to attend mandatory, regular and ongoing workshops to deepen their knowledge regarding cooperative learning implementation and other approaches that are effective in learning trigonometry.
- Regular workshops should be conducted on an ongoing basis to train teachers in the implementation of strategies for teaching and learning incorporating cooperative. It is suggested for the workshops to be conducted at the beginning of each term, before the teaching season commences in order to avoid the disruption of classes.
- Teachers should be awarded certificates of attendance to cooperative learning workshops in order to encourage them to value and encourage attendance on a continuous basis.

Recommendations for future research

This study was solely based on qualitative data. To overcome the weaknesses of a mono-method, it is recommended that future researchers follow mixed-method (qualitative and quantitative) approach. This will allow for the collection of in-depth data from interviews as well as numerical findings through survey questionnaires. Further research may also conduct a comparative approach to different classes in different learning districts in Limpopo. The benefit of this is the generalization of the findings to a bigger population.

5.5 LIMITATION OF THIS STUDY

This study extracted a sample of learners and a teacher and while the sample was enough to answer the predetermined research questions posed, the findings from this study cannot be generalised to Grade 11 learners and teachers in all the five Limpopo districts.

5.6 CONCLUSION

This study employed a qualitative method to investigate the use of cooperative learning to enhance conceptual understanding of trigonometry in Grade 11 mathematics classroom. A single case study was used as a research design to get an in-depth analysis and collect detailed data using semi-interviews and lesson observation of the cooperative learning of trigonometry in Grade 11 mathematics classroom from the learners and the teacher. Participants were purposely chosen and consisted of (n=30) Grade 11 mathematics learners and their mathematics teacher. Data from learners were collected using semi-structured interviews and through observation with the aid of an observation guide.

The findings from the study showed that the teacher did not highlight the importance of trigonometry to learners. The topic was only introduced, then the teacher continued with the lesson without explaining the importance of the topic in real life. Some learners contented that their difficulties in comprehending trigonometry were a result of teachers failing to teach in ways they understood.

In terms of cooperative learning, the study found that many learners were passively engaged, listening to or watching the teacher. Another observation was that learners rarely engaged in positive interactions with one another. Interaction among learners was moderate. At times, learners sat together but did not solve given tasks together. Literature findings concluded that other scholars whose classroom observations found that some learners reported difficulty in cooperating with other group members have observed such a state of affairs.

The steps of cooperative learning were reported to be unclear for learners. Teachers argued that the process had not been optimal, leading to less successful mathematical problem-solving process in trigonometry. Grade 11 teachers have some understanding of the characteristics of cooperative learning as well as the activities that underpin this approach to learning but they lack the skills to practically implement all the aspects of cooperative learning in the classroom setting.

This study concludes that more teacher training, the appointment of a curriculum advisor and the management of the volumes of trigonometry materials can create room for improvement in terms of engagement of learners during cooperative learning in trigonometry.

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APPENDICES

Appendix A: Semi-structured interview questions for learners

Interview guide: the use of cooperative learning in a mathematics classroom

The following are interview questions that learners will be asked

Name: ………………………… Number of years in grade ……………

Interview questions to learners:

1. What is your understanding of cooperative learning in a classroom?
2. How can you describe the best way of learning trigonometry in a mathematical classroom?
3. What is your understanding of the topic trigonometry? Do you find it hard? If yes, what do you think makes trigonometry difficult to understand?
4. Do you think using cooperative can improve your understanding of trigonometry than when a teacher method is used? Please elaborate.
5. What are the benefits of using cooperative learning in a mathematical classroom? Please name at least two.
6. Between basic ratios and reduction formula, which one do you find easier and why?

7. What mathematical resources do you use except the textbook for learning mathematics?
8. After the use of cooperative learning method to learn trigonometry, do you think your performance will change?
9. How did cooperative learning enhance your conceptual understanding of trigonometry?

Appendix B: Semi-structured interview questions for the teacher

Interview questions to a teacher:

1. What is your preferred method of teaching and why?
2. Do you regard trigonometry as a difficult subject?
3. What do you think makes the performance of learners in trigonometry poor?
4. What are the benefits of learning mathematics cooperatively to you as an educator?
5. If cooperative learning is used effectively, do you think it can enhance learners' conceptual understanding of trigonometry?

Appendix C: Observation guidelines

Observation guide in the mathematics classroom

Teaching and Learning Approach: Cooperative Learning			
1	2	3	4
Learners did not work as a group.	Learners worked as a group, but roles were not allocated to group members.	Learners worked as a group for an activity and roles were allocated to group members.	Learners worked as a group throughout the presentation and roles were allocated to group members.
COMMENTS			

Essence of Trigonometry: Basic ratios and reduction formula			
1	2	3	4
Trigonometry is not introduced.	Trigonometry fairly introduced with	The Use of Sin, Cos and Tan is	The lesson was well presented with Sin,

	revision activities from previous grade.	used in the introduction but with few activities to enhance learners understanding.	Cos and Tan as ratios, and relevant examples to enhance leaners understanding were given. The use of the mnemonic of SOHCAHTOA was used with enough examples.
COMMENTS			

Strength of Trigonometry: Application and Problem solving			
1	2	3	4
The importance of trigonometry was not highlighted.	The importance of trigonometry was highlighted by giving examples to describe real-life situation.	The importance of trigonometry was highlighted by giving examples to describe real-life situation and mathematical notations were used.	The importance of trigonometry was thoroughly highlighted with relevant real-life situations and mathematical notations and was integrated with science and engineering.
Comments			

Effectiveness in addressing or responding to trigonometric questions or problems			
1	2	3	4

Teacher is ineffective at helping students with problems or difficult task or questions.	Teacher generally tries to help students who approach problems or questions but is not consistently effective at addressing these problems.	Teacher help students who approach trigonometric problems effectively but with inadequate explanation.	Teacher is consistently effective in addressing students' questions, concerns and problems of all trigonometric topics: basic ratios and the use of reduction formula.
Comments			
Promoting learners' active engagement			
1	2	3	4
All the learners are not engaged in the lesson.	Most learners appeared distracted or disengaged.	Many learners were passively engaged, listening to or watching the teacher.	Most learners responded to teacher prompts, and/or actively manipulated materials.
Comments			

Promoting interaction among learners			
1	2	3	4
A teacher did not promote interaction among the students.	Students rarely engaged in positive interactions with one another.	There was an underlying positive tone to learners' interactions.	Learners were clearly positively connected to one another in planned activities to support their

			trigonometric understanding.
Comments			

Appendix D: Permission letter to the district official

Request for permission to conduct research at Matladi Project high school, Moletlane Circuit.

Title: Using cooperative learning in a grade 11 classroom to enhance conceptual understanding of trigonometry

Date ……………………………………

Department of Education Limpopo

Capricorn District Director

Dear ___________________

I, Rankweteke Puleng Edwin am doing research under supervision of Ngoepe M.G, a Professor in the Department of Mathematics Education towards a M.Ed. at the University of South Africa. We are inviting you to participate in a study entitled Using cooperative learning in a grade 11 classroom to enhance conceptual understanding of trigonometry

The aim of the study is to use cooperative learning to enhance conceptual understanding of trigonometry in Grade 11 mathematics classroom.

Your school has been selected because it has sufficient number of learners doing mathematics and your school environment is conducive for learning.

The I will be an observer for the trigonometry lessons for the period of three weeks in the second term as per the work schedule. A semi-structured interview will be conducted with the participants after the classroom observation at a time and the place that is convenient to both the participants and the researcher. The interview will be audio recorded for verbal transcription.

The benefits of this study are help teachers use cooperative learning effectively to enhance conceptual understanding of trigonometry in Grade 11 mathematics classroom.

There are no potential risks to participants. There will be no reimbursement or any incentives for participation in the research.

Feedback procedure will allowing any learner to contact the researcher to access the research results.

Yours sincerely

____________________________ (signature of researcher)

____________________________ (name of the above signatory)

Appendix E: LETTER REQUESTING ASSENT FROM LEARNERS

Title of my research is using cooperative learning in a grade 11 classroom to enhance conceptual understanding of trigonometry

Dear ___________ Date ________________

I am doing a study on Masters of Mathematics Education as part of my studies at the University of South Africa. Your principal has given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that your teachers can use to teach trigonometry better. This may help you and many other learners of your age in different schools.

This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

I would like to ask you to answer interview questions about the learning of trigonometry. Answering the interview questions will take no longer than an hour.

I will write a report on the study but I will not use your name in the report or say anything that will let other people know who you are. Participation is voluntary and you do not have to be part of this study if you don't want to take part. If you choose to be in the study, you may stop taking part at any time without penalty.. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. The benefits of this study are helping you to adapt and see the benefits of cooperative learning in the mathematics classroom. There are no potential risks in this study. You will not be reimbursed or receive any incentives for your participation in the research.

If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me at 061 482 7667. Do not sign the form until you have all your questions answered and understand what I would like you to do.
Do not sign the written assent form if you have any questions. Ask your questions first and ensure that someone answers those questions.

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school. I have understood the information about my study and I know what I will be asked to do. I am willing to be in the study.

__________________________ ______________________

Learner's name (print): Learner's signature: Date:

__________________________ ________________________

Witness's name (print) Witness's signature Date:

(The witness is over 18 years old and present when signed.)

__________________________ __________________________

Parent/guardian's name (print) Parent/guardian's signature: Date:

_________________________ __________________________

Researcher's name (print) Researcher's signature: Date:

Appendix F: Permission letter to the principal

Title: Using cooperative learning in a grade 11 classroom to enhance conceptual understanding of trigonometry

Date ……………………………………

The Principal

Matladi Project High School

Dear __________________

I, Rankweteke Puleng Edwin am doing research under supervision of Ngoepe M.G, a Professor in the Department of Mathematics Education towards a M.Ed. at the University of South Africa. We are inviting you to participate in a study entitled Using cooperative learning in a grade 11 classroom to enhance conceptual understanding of trigonometry

The aim of the study is to use cooperative learning to enhance conceptual understanding of trigonometry in Grade 11 mathematics classroom.

Your school has been selected because it has sufficient number of learners doing mathematics and your school environment is conducive for learning.

The benefits of this study are help teachers use cooperative learning effectively to enhance conceptual understanding of trigonometry in Grade 11 mathematics classroom.

There are no potential risks to participants. There will be no reimbursement or any incentives for participation in the research.
Feedback procedure will allowing any learner to contact the researcher to access the research results.

Yours sincerely

____________________________ (insert signature of researcher)

____________________________ (insert name of the above signatory

Appendix G: Permission letter to the teacher

UNISA college of education

The teacher

Matladi Project High School

Moletlane Circuit

Subject: Request to conduct a research in a school based in your circuit.

Dear sir/madam

My name is Rankweteke Puleng Edwin and I am doing research under the supervision of Ngoepe M.G, a Professor in the Department of Mathematics Education towards a M.Ed. at the University of South Africa. We are inviting you to participate in a study entitled Using cooperative learning in a grade 11 mathematics classroom to enhance conceptual understanding of trigonometry.

This study is expected to collect important information that could help teachers use cooperative learning to enhance conceptual understanding of trigonometry in Grade 11 mathematics classroom. My study will involve grade 11 learners and one grade 11 educators at Matladi Project Secondary School. All the data collected from the school will be analysed and reported regarding the study.

The information from this study will only be used for academic purposes. In my research report, and in any other academic communication, pseudonyms will be used. Collected data will be in my or my supervisor's possession and will be locked up for safety and confidentiality purposes. This research study is being carried out in the hope that it will contribute to the body of knowledge on how trigonometric concept could be well presented using cooperative learning pedagogy.

Kindly be informed of the following conditions of participation in the research study.

1. All participation is voluntary
2. The school's name will not be revealed in the findings of the research study
3. All discussions with participants will be treated with confidentiality
4. The school can withdraw from the research study at any time
5. If the school is willing to participate, it will kindly be requested to sign the consent form provided to it.

Signature of student

Name of student : Rankweteke P.E Supervisor: Prof Ngoepe M.G

Signature and name of the teacher : ………………………………………….

Appendix: Permission letter to Parents
Dear Parent

Your child is invited to participate in a study entitled Using cooperative learning in a grade 11 classroom to enhance conceptual understanding of trigonometry

I am undertaking this study as part of my masters research at the University of South Africa. The purpose of the study is to use cooperative learning to enhance conceptual understanding of trigonometry in Grade 11 mathematics classroom and the possible benefits of the study are the improvement of learners' performance in mathematics. I am asking permission to include your child in this study because he/she is a grade 11 mathematics learner. I expect to have 30 other children participating in the study.

If you allow your child to participate, I shall request him/her to (delete what is not applicable):

- Take part in an interview (explain procedures, when, where, time to complete survey)

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are good conceptual understanding of trigonometry which can lead to better mathematics performance. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities with the prior approval of the school and your child's teacher. In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty.

The benefits of this study are learners' increased knowledge of learning trigonometry with conceptual understanding.

There are no potential risks in this research study. There will be no reimbursement or any incentives for participation in the research.

If you have questions about this study please ask me or my study supervisor, Prof Ngoepe M.G), Department of Mathematics, College of Education, University of South Africa. My contact number is 061 482 7667 and my e-mail is dandyhunk@gmail.com. The e-mail of my supervisor is ngoepmg@unisa.ac.za . The Ethics Committee of the College of Education, UNISA, has already given permission for the study.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child:

Sincerely

Parent/guardian's name (print) Parent/guardian's signature: Date:

Researcher's name (print) Researcher's signature Date:

Appendix I: District's approval

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION
CAPRICORN SOUTH DISTRICT

CONFIDENTIAL

Ref: S4/2/1 Enq: ThobejaneMM: Contact: 015 633 9501 Email:khitadim@gmail.com

To: Mr. Rankweteke PE

From: District Director
Capricorn South District

SUBJECT: APPROVAL TO CONDUCT RESEARCH-MASTER'S OF EDUCATION DEGREE

1. Your letter dated 13/05/2019 refers.
2. It is with great pleasure to inform you that approval to conduct research as per subject above is hereby granted.
3. The research must be conducted in accordance with the department's policies and conditions such as but not limit to:
   - No disruption of Learning and Teaching
   - No publishing of research outcomes with privilege information before HOD's approval
4. We wish you best of luck in your studies. We believe this will add value to the education system in our province especially in Capricorn South District.

Kind regards.

District Director

17/05/2019
Date

Lebowakgomo Parliamentary Building, Unit B ~~Office No. 280, First Floor~~, Private Bag X03, Chuenespoort, 0745: Tel: (015) 633 9500, Fax 015 633 9900

The heartland of Southern Africa-development is about people

Appendix J: Ethics certificate

UNISA | university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2019/02/13

Dear Mr Rankweteke

Decision: Ethics Approval from 2019/02/13 to 2022/02/13

Ref: **2019/02/13/64148521/38/MC**
Name: Mr PL Rankweteke
Student: 64148521

Researcher(s): Name: Mr PL Rankweteke
E-mail address: dandyrunk@gmail.com
Telephone: +27 61 482 7667

Supervisor(s): Name: Prof Ngoepe
E-mail address: ngoepmg@unisa.ac.za
Telephone: +27 12 429 8375

Title of research:

Using cooperative learning in a grade 11 mathematics classroom to enhance conceptual understanding of trigonometry.

Qualification: M. Ed in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2019/02/13 to 2022/02/13.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2019/02/13 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

[illegible]

2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. No field work activities may continue after the expiry date **2022/02/13**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2019/02/13/64148521/38/MC** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof V McKay
EXECUTIVE DEAN
Mckayvi@unisa.ac.za

University of South Africa
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[illegible]
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Appwndix K: Interview transcripts and observation guide

Interview guide: the use of cooperative learning a mathematics classroom

The following are interview questions that learners will be asked

Interview questions to learners:

Researcher: What is your understanding of group work in a classroom?

S1 “I think group work is learning mathematics with other learners in the classroom,”

S2 “group work is to learn mathematics as a group of learners sitting on the same table and solving mathematics tasks together in a classroom”

S5 “to learn mathematics independently as classmates, and sitting in a group of 3-4 during the mathematics period.”

S7 “Group work for me means learning mathematics within a group with the other learners. In the group, we help each other understand the trigonometry questions that are being asked by the teacher and we help each other provide answers to the questions. This group work is different from when we are given tasks and told to solve them without the help of others. The group work means we have to interact with others and a lot of us contribute to the work that we are given as opposed to working as one person.”

S8 “sitting and solving mathematics activities in groups”

S9 “Sitting in groups during the mathematics period/lesson and learning from our classmate”

S11 “it is when learners help each other learn during the lesson in a classroom.”

Researcher: What are your challenges in learning and understanding trigonometry?
S1 "Trigonometry requires a lot of understanding, especially when you are required to simplify, so I get activities which want us to simplify very difficult and they are given lot of marks"

S2 "The ability to solve for x, and factorise is a requirement to solve trigonometric problems, I always have to remember how I solve algebra questions to be able to solve trigonometry activities"

S5 "The use of sin, cos and tan makes trigonometry complicated and the understanding of the relationship among that is very important"

S7 "The way teachers teach mathematics is not good because we are passing other subjects, and the lack of resources we use in the classroom, for example sir we only use one textbook, the school don't give us study guides or help us watch videos of the topics which are difficult for us"

S9 "Lack of practice and understanding, trigonometry is there in grade 10 and we were not well taught and it becomes a problem when it continues in grade 11, some activities will need us to remember the basics but it is always a problem to remember because we were not taught good in grade 10"

Researcher: Which method of teaching and learning do you think is the best for learning trigonometry for understanding?
S1 "I think group work can be better is we are always using it because we will learn from each other as learners"

S2 "when a teacher is giving a lot of examples and activities to do in the classroom"

S5 "when learners are writing the solutions on the board and explaining how they got their answers step by step."

S7 “the method that I think can be best is when learning with classmates, sitting together on a table and solve given mathematics questions”

S9 “Using various resources to help understand trigonometry better, like study guides, previous questions papers and watching you tube videos.”

Researcher: What are the benefits of using cooperative learning in a mathematical classroom? Please name at least two.

S1 “learning from each other and having time to share ideas with your classmates”

S2 “Having the confidence to share ideas with classmates, sometimes it is not easy to ask a teacher how he got an answer, but it is easier when you can ask a fellow classmate, so that they can explain and you can change views and learn from each other”

S5 “I think the benefit is being able to present answers to the whole class and explain how you got your answers,”

S7 “Learning to rectify each other and accept our classmates’ inputs on solving mathematics problem”

S9 “Learning independently as learners without the teacher spoon feeding us, as learners we should do the activities together so that we can we can be able to answer them when writing a test or exam.”

Researcher: What do you find difficult in solving trigonometric problems/questions?

S1 “The challenge that I face with learning trigonometry is that the subject requires me to have a lot of understanding. This is especially in my case when the problems given by the teacher require me to simplify. It has taken me a while to grasp that but the more I get confused with simplifying, the more it creates problems for me during homework and tests."

S2 "I think my difficulty is actually with teachers. I do not think the way they teach is enough because no matter how much information is explained to me; I am still left confused. I think that we lack more resources that can make learning trigonometry easier for us. We need more materials to help us understand trigonometry better so that if I do not understand what the teacher is saying, I can go to those materials and try to see if they can make me better understand. Those materials may be videos, maths props and textbooks that have good guides to explain step by step how to solve trigonometry problems"

S4 "Applying basic algebra to factorise or solve trigonometric ratio question and being able to solve many activities in a homework activity since our teacher is giving us a homework to do every day the maths period"

S5 "not able to use a diagram when solving for questions with basic ratios, not able to draw the Cartesian plane to present information and solve the given problems, our teacher always says it is best to present information on the Cartesian plane since it can help making it easier to solve given activities"

S6 "I have a big problem using $\sin\theta$, $\cos\theta$ and $\tan\theta$. I think they make trigonometry complicated for me very much because it is just a lot of things to learn and then keep in mind because you need to have that knowledge every time. This makes it difficult every time I am given assignments or tests. During exams, I have to study this and spend most time on it because it is so challenging and I need to go into the exam knowing how to solve the problems. I try to study how the three relate because I think the understanding of the relationship among that is very important"

S7 "Lack of practice and understanding. This is because trigonometry is there in Grade 10 but we were not well taught. This is one of the greater reasons why understanding trigonometry becomes a problem when it continues in Grade 11"

S9 “to convert trigonometric expression using the CAST diagram, and the not understanding the special angles, using a calculator even if the question says without the use of a calculator because I will not be understanding how to solve without the help of a calculator”

S11 “I have a problem with solving for x. This is a problem for me because questions such as that are asked in the class all the time and I end up feeling discouraged and sometimes frustrated because I do not know how to solve those problems. I also have a difficulty when I am asked to factorise because I cannot easily follow what is being asked of me. In these two issues with finding x and factorising, I just end up confused and failed in assignments and test. The knowledge of solving for x and to factorise helps with solving trigonometric related tasks”

Researcher: What mathematical resources do you use except the textbook for the learning of mathematics?

S1 “I use study different guides that I buy since they don’t give us at school”

S2 “our teacher provide us with previous question papers to prepare for the test”

S5 “Pamphlets from the teacher during and before the lesson”

S7 “mathematics workbook”

Researcher: How did cooperative learning enhance your conceptual understanding of trigonometry?

S1 “We hardly use group work in the classroom, it was very helpful as I was able to learn from my group mates, I think our mathematics and physics teachers should normalise teaching us in groups because these subjects need a lot of practice”

S2 “I was able to express myself when solving problems, and I was able to see my mistakes because mathematics is difficult especially trigonometry, but some learners in the groups will just be quiet and not share their ideas or contribute to the task given to the group, so that was not good at all"

S5 "It built my self-confidence, as I was able to comment on the work written on the board by other groups and I was able to improve my solving skills through their explanation and the way they were solving given exercises"

S7 "Trigonometry has been difficult for me in grade 10, but this time using group work made me understand the trigonometric ratios and reduction formula to simplify trigonometric questions"

S9 "Learning in groups need enough time and space, the class should not be overcrowded and our teachers should have effective ways to facilitate cooperative learning."

Interview guide: the use of cooperative learning a mathematics classroom

The following are interview questions that teachers will be asked

Name: XXX Number of years Teaching grade: 24

Interview question to a teacher:

Researcher: What is your preferred method of teaching and why?

"Teaching methods should be mixed, because learners learn differently, the methods should be mixed during the process of teaching, but I mostly recommend group work, learners should investigate when given activities to do, I shouldn't always spoon feed them."

Researcher: Do you regard trigonometry as difficult subject?

"No, trigonometry is the easiest since it has 3 trigonometric ratios (Sin, Cos and Tan) in Grade 11 and 12, learners view at as a difficult topic because they don't learn the laws and procedures that needs to be followed in order to understand and solve all given activities with understanding, for example when we work with reduction formula learners should understand how the circle works, when learners can understand on the first quadrat all trig ratios are positive and so forth"

Researcher: What do you think makes the performance of learners in trigonometry poor?

"Lack of practice, most learners will seem to be understanding during the process of teaching and learning but performing very poor during the tests and examination. The poor performance in mathematics as a whole is also because these learners don't have study groups (for mathematics practice and other subjects). Some learners bunk classes or are absent most of the times and that result in them missing a lot of important lessons, some learners don't help each other i.e. some learners are just selfish"

Researcher: What are the benefits of learning mathematics in a cooperatively to you as an educator?

“I believe learners learn best from each other, and using group work they are able to rectify each other, learn from each other and to encourage each other to learn for understanding. Learners can help each other in a sense that the other learner might be good in a certain subject e.g. accounting and the other in mathematics, so these learners can benefit from each other”

Researcher: If cooperative learning is used effectively, do you think it can enhance learners’ conceptual understanding of trigonometry?

“Group work consumes times, since learning will need enough time to discuss, work together and present their work to the whole class. The classrooms are mostly overcrowded with at least 50 learners in one classroom, and that makes it hard for learners to sit in groups. The government should intervene for the maximum of 30 learners to be in one classroom to make learning conducive: allowing other teaching methods to be easily implemented. And teachers should be workshopped about other teaching strategies like cooperative learning for these methods to be effectively and well used in the classroom for the betterment of learners’ performance.”

Researcher: Do you think using cooperative learning can improve learners’ understanding of trigonometry than when a traditional textbook method is used? Please elaborate?

“We teachers use different method of teaching, but like I said earlier I believe in mixing these teaching methods to enhance understanding of learners in mathematics in general. I use all the methods of teaching in my classroom but cooperative learning is better than traditional textbook method since it is learner- centred, it, makes learners independent of their own learning.”

Observation guide in the mathematics classroom

TEACHING AND LEARNING APPROACH: COOPERATIVE LEARNING			
1	2 **X**	3	4
Learners did not work as a group	Learners work as a group, but roles were not allocated to group members.	Learners work as a group for an activity and roles were allocated to group members.	Learners work as a group throughout the presentation and roles were allocated to group members.
COMMENTS	Learners were sitting in groups but the cooperative learning wasn't effective as the teacher didn't allocate any roles to them.		

Essence of Trigonometry: Basic ratios and reduction formula			
1	2	3	4 **X**
Trigonometry is not introduced	Trigonometry fairly introduced with revision activities from previous grade	The Use of Sin, Cos and Tan is used in the introduction but with few activities to enhance learners understanding.	The lesson was well presented with Sin, Cos and Tan as ratios, and relevant examples to enhance leaners understanding were given. The use of the mnemonic of SOHCAHTOA was

			used with enough examples.
COMMENTS	The lesson delivery on trigonometry was well done, the teacher is knowledgeable and confident about the content of trigonometry.		

Strength of Trigonometry: Application and Problem solving.			
1 X	2	3	4
The importance of trigonometry was not highlighted.	The importance of trigonometry was highlighted by giving examples to describe real-life situation.	The importance of trigonometry was highlighted by giving examples to describe real-life situation and mathematical notations were used.	The importance of trigonometry was thoroughly highlighted with relevant real-life situations and mathematical notations and was integrated with science and engineering.
Comments	The topic was just introduced and then the teacher continued with the lesson without explaining the importance of the topic in real life.		

Effectiveness in addressing or responding to trigonometric questions or problems			
1	2	3	4 **X**
Teacher is ineffective at	Teacher generally tries to help	Teacher help students who	Teacher is consistently

helping students with problems or difficult task or questions	students who approach problems or questions but is not consistently effective at addressing these problems.	approach trigonometric problems effectively but with inadequate explanation	effective in addressing students' questions, concerns and problems of all trigonometric topics: basic ratios and the use of reduction formula.
Comments	The teacher was addressing learners' questions in a satisfying manner at all times.		

Promoting learner's active engagement			
1	2	3 **X**	4
All the learners are not engaged in the lesson	Most learners appeared distracted or disengaged	Many learners were passively engaged, listening to or watching the teacher.	Most learners responded to teacher prompts, and/or actively manipulated materials.
Comments	There is a room for improvement in terms of engagement of learners in a lesson by the teacher.		

Promoting interaction among learners

1	2 **X**	3	4
A teacher did not promote interaction among the students	Students rarely engaged in positive interactions with one another	There was an underlying positive tone to learners' interactions	Learners were clearly positively connected to one another in planned activities to support their trigonometric understanding
Comments	Interaction among students was moderate: Sometimes learners would sit together but not solve given tasks together.		